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A DIFFERENTIAL COLOR MIXER WITH STATIONARY DISKS

BY PAUL THOMAS YOUNG

University of Illinois

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I. Introductory Statement

The Maxwell [1] color wheel has been modified by Marbe [2] so that the angular ratio of *two* color-disks may be varied during rotation.[3] With the differential color mixer constructed by the writer *several* disks may be used simultaneously, since the disks themselves are stationary. Light reflected from the disks is mixed by means of a rotating prism or mirror.

The arrangement is somewhat similar to that described by Brodhun.[4] He pictures two rhombs placed so that the emergent light and the incident light lie along the same straight line. These rhombs are rotated together about a common axis and between them are placed variable double-sectored disks. Brodhun's apparatus is intended for photometric work, and it uses transmitted light.

The only direct attempt to build a differential color mixer along the lines of the present apparatus, so far as the writer is aware, was made a number of years ago by President E. C. Sanford, of Clark University. An account of his work has never been published.[5]

[1] Maxwell, J. Clerk, *Transactions of the Royal Society of Edinburgh*, XXI., Pt. 2, 1857, 275–99 (paper read 19 March, 1855). On p. 298 Maxwell pictures his well-known color mixer—originally a hand-spun top.

According to Titchener (*Exper. Psychol.*, Vol. I., Pt. 2, 1910, 14–16) color mixing by the method of rotating disks was originated by P. van Musschenbraek in 1768. It was employed by J. Plateau in 1853, and finally perfected by J. Clerk Maxwell in 1857.

[2] Marbe, K., *Centralblatt f. Physiol.*, 10 März, 1894; 23 März, 1895. Also *L'Année Psychol.*, II., 1896, 782–5; V., 1899, 391–3.

[3] The psychological laboratory of the University of Illinois possesses a demonstration color mixer modeled after Marbe's apparatus. It is pictured and described by Ruckmick, C. A., 'New Laboratory Equipment,' *Amer. J. of Psychol.*, 27, 1916, 538–40.

Professor S. W. Fernberger, of the University of Pennsylvania, recently demonstrated to the American Psychological Association a more substantial form of the apparatus.

Brodhun has described a photometer very similar in principle to the Marbe color mixer. Brodhun, E., 'Rotierender Sektor, dessen Winkel während der Rotation verändert und abgelesen werden kann,' *Zeitschrift für Instrumentenkunde*, 24, 1904, 313.

[4] Brodhun, E., 'Photometrie.' A. Winkelmann, 'Handbuch der Physik,' 6, Optik, 747–84.

[5] President Sanford has kindly permitted me to quote part of a letter, dated August 13, 1921, wherein he describes this apparatus.

The apparatus devised by the writer resembles President Sanford's in that the colored disks are stationary and mixing is accomplished by rotating prisms or mirrors between the disks and the eye. Stationary disks [1] have several advantages over rotating. First, several disks may be used simultaneously and the ratios changed *ad libitum* without stopping the apparatus. Second, one set of disks may be removed and another substituted without loss of time. Series of disks for

"The plan which I followed some fifteen years or so ago is all right in principle but involved practical difficulties—one in particular—which had not been removed when I gave up the laboratory for administrative work. For that reason I have never described it in print, though I demonstrated the apparatus once at a meeting of experimental psychologists at Cornell.

"The plan is roughly indicated below (see Fig. A).

"The prism is a 90° prism. The dotted line shows the course of the rays of light, which are totally reflected from the face *AB* and leave the face *AC* in precisely the direction in which they entered *BC*. The prism revolves around the line connecting the eye with the center of the disk. One actual rotation of the prism gives two apparent rotations of the disk—as I judge would be the case with your mirrors.

"The snag which I was not able to get away from when I was at work at the apparatus was the adjustment of the prism in such a way as to produce a perfectly centered (apparent) rotation of the disk. I could get something pretty good but not perfectly satisfactory. The center of the disk described a curve something like this (President Sanford here pictured a trochoidal curve with two loops, a smaller loop enclosed within a larger). Whether still more careful adjusting of the prism would correct this, or whether there was some defect in the grinding of the prism, or whether something else was the matter I do not know."

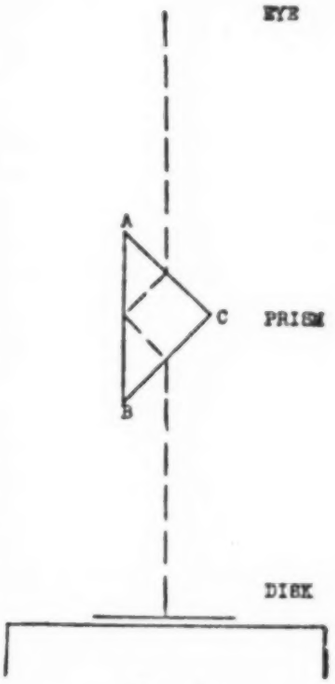

FIG. A

[1] An apparatus which depends upon pigmented surfaces has its limitations. There seems to be a rather general agreement among investigators that colored papers are useless for precision work unless the exact composition of the reflected light can be determined. Even then colored surfaces change with time. For a recent statement see: Jones, L. A., and Reeves, P., 'The Physical Measurement and Specification of Color,' *Psychol. Rev.*, 1920, 27, 453–66.

For statements regarding various methods of color mixing the reader is referred to:

Tigerstedt, R., 'Handbuch der physiologischen Methodik,' Band III., 1, Sinnesphysiol. II., 63–80.

Luckiesh, M., 'Color and Its Applications,' 1915, 60–8.

Maxwell, J. Clerk, *op. cit.*, 288–293.

Titchener, E. B., *op. cit.*, 14–16.

testing, demonstration, or for work with constant stimuli may be made up permanently. As soon as a disk is placed before the mixer the reflected light is instantly mixed by the rotating apparatus. It is not necessary that the disks be centered for an approximate centering—with our apparatus —gives a uniform color field.

The movable part of the color mixer in its original form consisted of two parallel mirrors arranged as in Plate III. to rotate between the eye of the observer and a set of stationary disks. Later the mirrors were abandoned and two 90° prisms substituted. Finally a single rhomb [1] was adopted. (See Plates I. and II.) The mirrors, prisms and rhomb are equivalent in the reflection of light. (See Figs. 1 and 3.)

When the original model was tested it was discovered that the ordinary Maxwell disks failed to give a uniform color field. Two disks, for example, set at 180° White and 180° Black gave a gray which showed gradations from relatively light to relatively dark. The reason for this may not be immediately obvious, but it will be clear from the mathematical discussion below (Propositions I. and II.).

The discovery that double-sectored disks give a uniform field came as a happy accident after several months of study upon the problem. (See the disks in Plates II., IV., V., VI., and especially VII.) It is essential that double-sectored disks [2] be used along with the rotating apparatus.

During the four years that the writer has been experimenting with differential color mixers five or six arrangements of rotating mirrors and prisms have been tested. As a result two have survived and these will be designated as Type I.

[1] The rhomb is 3 x 3 cm. in cross section and the length of the upper and lower surfaces is 4 cm. The angles are 45° and 135°.

[2] The principle of the double-sectored disk is not new. Double sectors are used, for example, in the Aubert episkotister; and they are commonly employed in other sorts of photometric apparatus to cut down the intensity of light. See:

Helmholtz, H. von, 'Handbuch der Physiologischen Optik,' 1911, II., 105.

Stine, W. M., 'Photometrical Measurements,' 1919, 24.

Abney, W. deW., 'Researches in Colour Vision,' 1913, 69.

Luckiesh, M., *op. cit.*, 64.

The special type of double-sectored disk which uses reflected light is new, so far as the writer is aware.

and Type II. The first type (Plates I., II. and III.) is recommended for experimental work. The second type (Plates IV., V., VI.) is useful for certain demonstrations.[1]

The apparatus will be better understood from the following description of the plates, and from the mathematical considerations.

II. Description of the Plates

Type I.—Plate I. shows the latest form of the apparatus. A hollow shaft, one inch in diameter, is mounted upon two sets of ball bearings. Upon the shaft is a pulley with a V-groove. A leather belt connects this pulley with the driving motor. Upon one end of the shaft is an aluminum disk which carries a doubly reflecting rhomb and also a counter-weight to prevent vibration during rotation. A discussion of the geometrical optics of this device is given below.

Plate II. shows the apparatus set up for operation. The rotating rhomb is placed between the eye of the observer and a set of stationary colored disks. Light from the colored disks is doubly-reflected by the rhomb so that it passes through the hollow shaft to the eye of the observer. When the rhomb is rotating, light from the disks is mixed and the observer sees a uniform color field.

Plate III. shows the original form of the apparatus. Two parallel mirrors are attached at an angle of 45° to an aluminum disk. A counter-weight is added to prevent vibration during rotation.

Type II.—At the right of Plate IV. is shown an apparatus for rotating a plane mirror (diameter 3 inches). The mirror is adjusted so that its plane is at an angle of 86° to the axis. The mirror is attached to a solid aluminum base and this base is pivoted in an aluminum cup in such a manner that the plane of the mirror may be tilted. Two thumb-nuts lock the mirror to the cup at the desired angle.

The original form of this type of apparatus is shown in the center of Plate IV. Here the base supporting the mirror

[1] The mirror in Type II. gives a lustre to the colors which, so far, we have not been able to eliminate completely. Another difficulty with Type II. lies in the illumination of the disks. In a dark room it is necessary to direct the artificial light towards the observer's eyes. Various screens are necessary.

is pivoted directly to the axis of the motor. This construction proved to be too light for service, but there is nothing inherently wrong with the principle.

Plate V. shows the set-up for color mixing. Immediately in front of the observer are the stationary double-sectored disks mounted upon an aluminum frame. The front view of the disks is pictured in Plate IV. In the center of the disks is a hole. Light is reflected from the disks to the mirror and then through the central opening to the eye of the observer. When the mirror rotates the light reflected from the disks is mixed.

Plate VI. shows the arrangement of the apparatus for demonstrating color mixture to a small group of students. It is necessary to direct the attention of the observers to an oval area of the mirror image within which the color field is uniform. The mirror image outside of this uniform area shows many color gradations and patterns. This arrangement does not give, theoretically, a uniform color field. Practically, however, lack of uniformity is not observable when the plane of the colored disks is perpendicular to the axis of the apparatus and when the disks are not greatly removed from the central axis.

Plate VII. gives a front view of a set of demonstration disks, eighteen inches in diameter. Each color of the set has two disks and the two are bound together so that they may be moved by a single handle. By shifting the handles it is possible to set up almost immediately any desired combination of color components. The present set of disks is made from Hering's papers. The white disk is extended beyond the others and calibrated in degrees, so that any particular setting may be read off and recorded directly and immediately.

Plate VIII. shows a mirror image taken with the apparatus in rotation. The image is a projection of a short strip of white card upon black velvet; and it appears to the eye as a hollow cone which stands out clearly in three dimensional space.

III. Mathematical Considerations: Type I

The first type of color mixer is diagramed in Fig. 1. $ABCD$ is a rhomb; AD and BC are the parallel reflecting surfaces. This rhomb is so mounted on the apparatus that

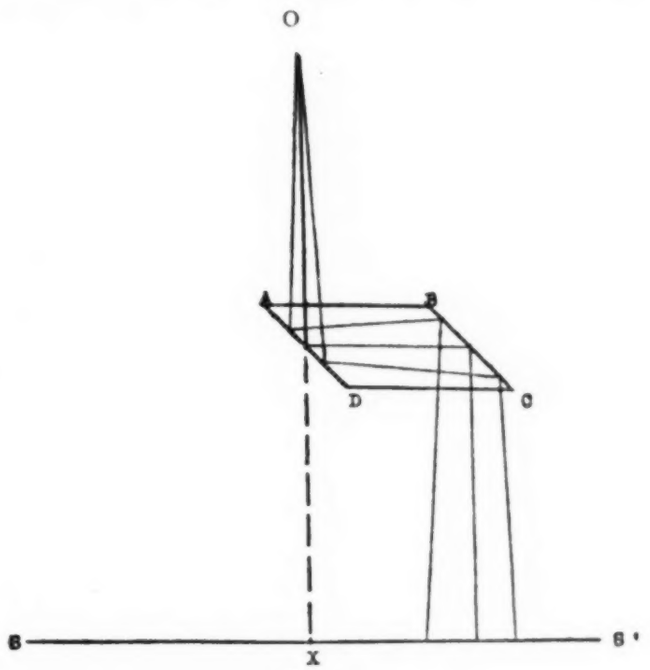

Fig. 1

it may be rotated about the axis OX. Light waves reflected from the colored disks SS' (represented in cross-section) are doubly reflected by BC and AD to the eye O.

The demonstration that this type of apparatus gives a uniform color field when used in conjunction with double-sectored discs rests upon two propositions. These propositions are given below.

Proposition I.—Fig. 2 represents two straight lines, AB and CD, intersecting at a point, O. The shaded areas between one pair of vertical angles are intended to represent black; and the unshaded portions, white. The proposition to be proved is the following: If from any point, O', a circle be drawn so as to include O within its circumference, the total number of degrees of black in this circumference is equal to $< AOC + < DOB$, and the total number of degrees of white is equal to $< AOD + < COB$.

Proof: Let E, F, G, H mark the points where the circumference cuts the lines AB and CD. Through E draw a line parallel to CD cutting the circumference at I. Draw IO' and $O'G$.

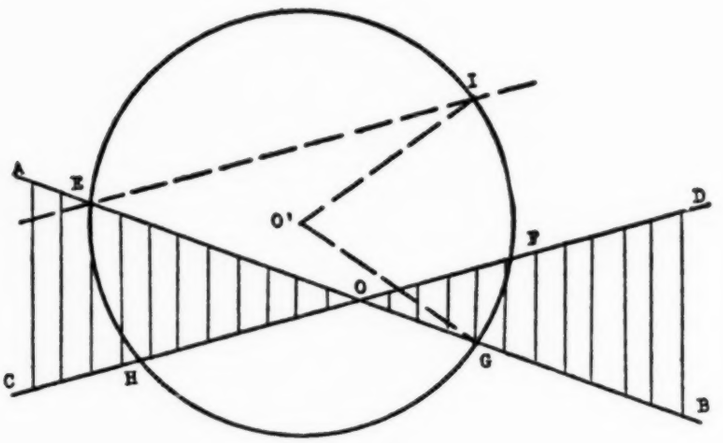

FIG. 2

Now arc IF = arc EH, since parallel lines cut equal arcs. Hence $< IO'G$ = total number of degrees of black in the circumference; and $360° - < IO'G$ = total number of degrees of white.

The central $< IO'G$ = twice the inscribed $< IEG$. But $< IEG = < EOC = < BOD$, since EI is parallel to CD. Therefore, $< IO'G = < AOC + < BOD$.

Further since

$$< AOD + < BOC = 360° - (< AOC + < BOD)$$

it follows that the circumference contains white equal in degress to $< AOD + < BOC$.

Corollary I.—Every circle which includes O contains in its circumference a constant number of degrees of black and a constant number of degrees of white.

Corollary II.—If three or more lines intersect at a point, and if each pair of vertical angles includes the same color, then every circle including the intersection point within its circumference contains the same number of degrees of the color components.

Proposition II.—With the rotating apparatus previously described every fixed point in the field of vision corresponds, during rotation, to a circle upon the colored disks.

Light is actually reflected from the colored disks, *SS'*, Fig. 3, through the rotating apparatus to the eye. In order to determine what path upon *SS'* corresponds, during rotation, to a fixed point in the apparent field of vision let us project a beam of light from the eye, *E*, through the apparatus and to the surface, *SS'*. Let the beam *EF* represent any ray of light parallel to the axis of rotation, *OX*. This beam is doubly reflected by the surfaces *AB* and *CD* over the course *EFGH*. *H* is the point at which the beam strikes *SS'*.

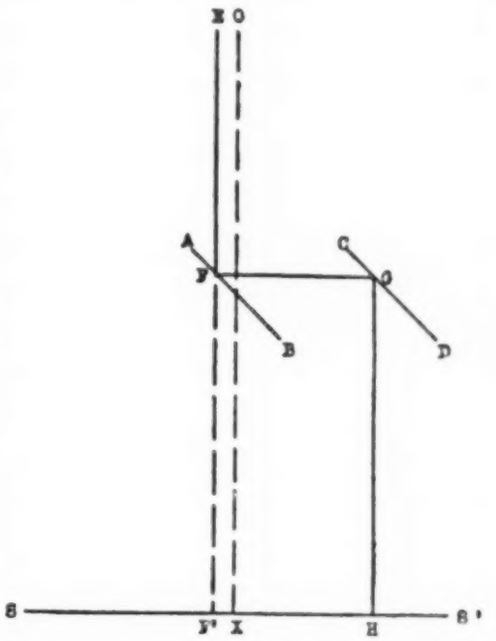

FIG. 3

It follows from the laws of reflection that *FG* is parallel to *SS'* when *AB* and *CD* are at an angle of 45° to *OX* and when *EF* is parallel to *OX*. This is true regardless of the position of *F* upon *AB*. Hence the rotation of the apparatus cannot change the distance *FG*. Rotation changes only the horizontal angular position of *FG* and the vertical position of *F*.

Extend *EF* to *F'*, a fixed point upon *SS'*. *FGHF'* is a rectangle. Hence *F'H* is constantly equal to *FG*, and *H* describes a circle about *F'* as a center. The angular velocity of *H* about *F'* is simultaneously equal to the angular velocity of the apparatus about *OX*.

Some beams of light striking the eye are not parallel to *OX* but make a slight angle to it (see Fig. 1). If *EFF'*, Fig. 3, be tilted a bit so that it is not perpendicular to *SS'*, the path of *H* is not a circle but an oval. This means, in theory, that the field of vision is not strictly uniform except when all the beams reaching the eye are parallel.

Practically, however, no lack of uniformity in the color field has been detected. Any irregularity due to the above-

mentioned source of error is subliminal. As the eye is withdrawn from the rotating apparatus this error, regarded theoretically, becomes less and less. It would be possible, we believe, to modify the apparatus so as to eliminate entirely this error—should it ever be found necessary to do so.

IV. Mathematical Considerations: Type II

A. The Mathematical Optics of the Plane Mirror Apparatus (Centered).—In Fig. 4, MN represents a plane mirror tilted to the axis of rotation, OX', so that the angle MXO is approximately 86°. SS' represents a cross-section of the color-disks which face the mirror. In the center of SS' is a circular opening. The eye of the observer is at O.

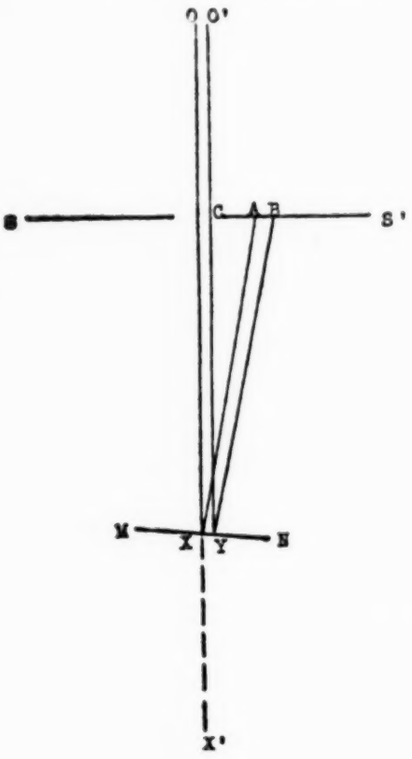

Fig. 4

In order to determine what path upon SS' corresponds, during rotation, to a fixed point in the field of vision let us proceed as above, projecting beams of light from the eye through the apparatus to the color-disks.

Consider first the path of a beam of light, OXA, where OX is identical with the axis of rotation. The angle OXA is a function of the angle MXO; and both of these angles are constant during the rotation of the apparatus. $\angle OXA = 180° - 2\angle MXO$. The point A is a constant distance from OX and hence describes a circle upon SS'. From the laws of reflection it follows that AX is in a plane perpendicular to the surface MN. This perpendicular plane rotates about OX with an angular velocity simultaneously equal to that of the mirror, MN, about the axis, OXX'. In other words the horizontal angular velocity of A is constantly equal to that of the rotating apparatus, and the path of A is circular.

Consider next the path of a beam of light, $O'YB$, where

O' is any point in the field of vision and $O'Y$ is parallel to OX. Let C be the point where SS' (extended) cuts $O'Y$.

$< O'YB = < OXA$ since $O'Y$ and OX are parallel. $< O'YB$ lies in a plane which is, during rotation of MN, constantly perpendicular to MN. Since this plane is always parallel to the plane determined by OXA, or identical with it, the angular velocity of BY is simultaneously equal to that of the rotating apparatus.

The path of B is not quite circular, but it is slightly oval, for the reason which follows. The radius $CB = CY \tan < CYB$. The leg CY varies in extent slightly with the angular position of MN relative to OX'. Hence the leg (radius) CB is not quite constant.

Practically, with the apparatus used by the writer, the field appears uniform. The theoretical variation from uniformity is actually subliminal.

If the projected beam of light makes an angle to the axis of rotation, its path upon SS' is not circular. This case will be discussed presently.

B. The Mathematical Optics of the Plane Mirror Apparatus (Not Centered).—Fig. 5 shows an arrangement of the apparatus

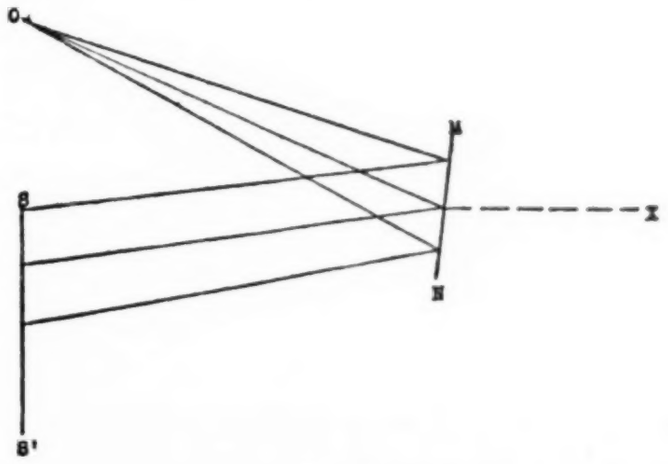

FIG. 5

in which every beam of light makes an angle to the axis of rotation. (See Plate VI.) The colored disks are placed at SS' and the eye above them at O. It is important to determine (1) the appearance of the mirror image of SS' seen in

the rotating mirror from O, and (2) the path upon SS' of a beam of light projected from O at a constant angle to the axis X.

1. *The Image of a Point with a Rotating Plane Mirror.*—Before considering the image of a fixed point, when viewed in a rotating mirror, let us remind ourselves of a well-known principle of reflection. A beam of light, PR, Fig. 6, striking a reflecting surface, MN, at any angle PRM is reflected so that the angle of reflection is equal to the angle of incidence. If the eye be at O, the image of P is localized at a point, P'. The line PP' is always perpendicular to the surface MN and always bisected by this surface. Hence in determining the image of a fixed point, seen in a rotating plane mirror, it is necessary to find the path of P'—remembering that PP' is perpendicular to MN and bisected by it.

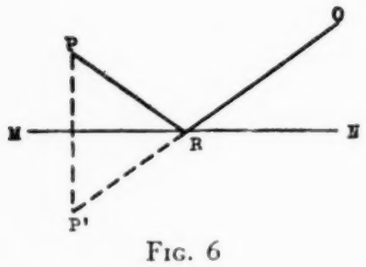

FIG. 6

Figure 7 shows, in cross-section, the plane mirror in two positions 180° apart—MN and $M'N'$. RX is the axis of rotation.

When the mirror is in the position MN the image of a point, P, is indicated by P'; and when the mirror is in the position $M'N'$ the image of P is at a point indicated by P''. The distance $P'R$ = the distance PR; also $P''R = PR$. Regardless of the angular position of the mirror relative to RX the image of P is at a distance from R equal to PR. Hence, the image of P lies upon a sphere the radius of which is PR.

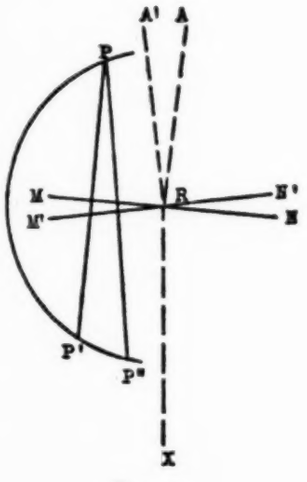

FIG. 7

At R erect a line AR perpendicular to the surface MN. This perpendicular is seen at two positions 180° apart, AR and $A'R$. The line AR generates a cone during the rotation of the mirror and the angle at the apex of the cone depends directly upon the angle at which the mirror is set relative to RX. The angle at the apex equals $180° - 2 < NRX$. Since PP' is perpendicular to the mirror it is con-

stantly parallel to AR. From this it follows that PP' generates a cone during the rotation of the mirror and the angle at P is equal to the angle ARA'.

Therefore, the image of a point, P, is a quartic curve formed by the intersection of a cone and a sphere. In any particular case the constants necessary to determine the equation of this curve may easily be measured.

Having determined the image of a point, it is possible to determine the images of lines and surfaces. (See the photograph of the projection of a line, Plate VIII.) The rotating mirror apparatus makes visible exceedingly complex projections. The mathematical discussion of these complex projections is beyond the scope of the present paper.

2. *The Path of a Beam of Light Projected at a Constant Angle to the Axis of Rotation.*—In Fig. 8 let OX represent a

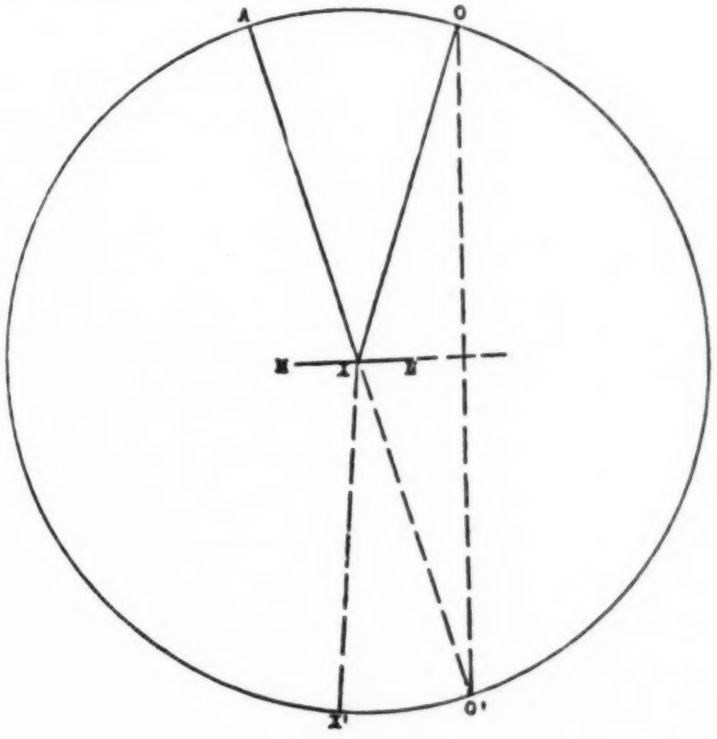

FIG. 8

beam of light projected at a constant angle to the axis of rotation XX' and striking at the central point X. This beam is reflected over the path XA.

Draw OO' perpendicular to the mirror surface MN and

extend AX to O'. O' may now be regarded as the source of the beam of light. The beam of light AXO' passes through the fixed point X. Its movement is, therefore, determined by the movement of the point O'.

By a proof identical with that above for determining the image of a point it can be shown that O' is always upon the surface of a sphere which has OX for its radius; and O' is also upon the surface of a cone generated by OO' and having O as its apex, and an angle at the apex equal to

$$2(90° - < MXX').$$

From the above considerations it appears that the path of A (upon a spherical or plane surface) is circular only when OX is identical with the axis of rotation. A very close approximation to a true circular path results when the angle OXX' approximates 180° and when the angle MXX' approximates 90°.

If the point X is not at the center of the mirror, the movement of XA is complicated by a slight vertical movement of X which takes place during rotation. This source of error was met in the discussion of Fig. 4.

The arrangement shown in Plate VI, therefore, is not recommended for colorimetric work. It is only an approximation. The value of this arrangement is for demonstration.

V. Practical Considerations

A. The Construction of Double-Sectored Disks.—(1) If ordinary colored disks be used, it is necessary to have two for each color in the mixture and to arrange them in double sectors. Figure 2 shows a diagram of the arrangement for black and white. Plates II., IV., V., VI. and VII. show pictures of the arrangement necessary with double-sectored disks, using several colors.

Obviously it is highly desirable to bind together every pair of colors so that their visible edges are permanently held 180° apart, and so that each pair may be moved as a unit. The method of accomplishing this end is given in the following instructions.

Cut the disks in the regular manner but leave handles projecting outward as pictured in Fig. 9. The handles should be placed so that two disks of the same color may be threaded and bound through their handles with their slits exactly 180°

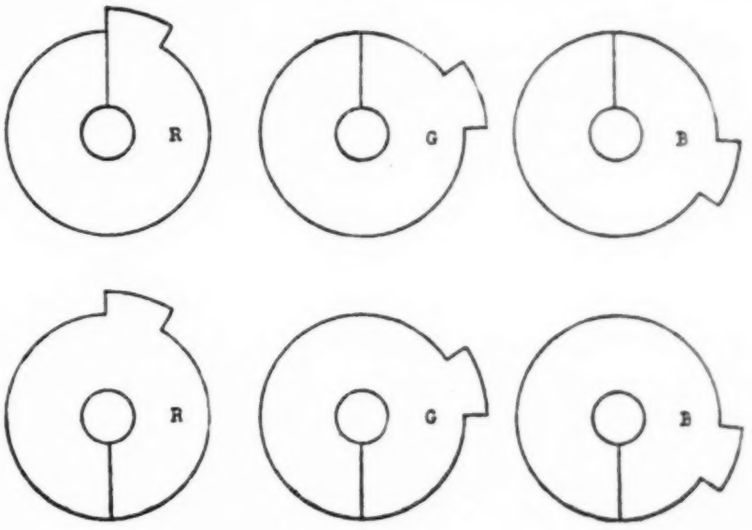

FIG. 9

apart. Fig. 9 shows two red (*R*), two green (*G*), and two blue (*B*) disks prior to threading and binding. Before binding the disks several thicknesses of cardboard should be placed between the handles in order to make room for disks which must pass between. A paper binder run through handles and cardboard will hold both disks together. A little mucilage may be used advantageously.

The number of degrees in the handle is a matter of convenience; 20° is ample. When several disks are to be used simultaneously all of the handles must be included in 180° and the handles must not overlap. Fig. 9 shows the correct placement of the handles for three sets: *R*, *G*, and *B*. If *n* represent the number of colored disks to be used simultaneously, then $180°/n$ is the maximum number of degrees per handle.

The diameter of the disks is also a matter of convenience and it should depend upon the nature of the apparatus with which the disks are to be used. The eighteen inch disks shown in Plates VI and VII are good for demonstration to small groups of students, but the writer has never used disks of

this size for any other purpose. With the latest form of the apparatus, shown in Plates I. and II., disks of 15 cm. are used.

The writer has found that ordinary colored papers are not satisfactory for making double-sectored disks. They curl, catch and tear. Upon the Maxwell color mixer papers lie flat against the cardboard base when rotated, but with stationary disks the situation is different. Curling gives rise to a shadow which disturbs the uniformity of the color field. The chance of disks catching and tearing is increased when double the ordinary number is used.

Thin stiff cardboard and heavy papers have been found to be more satisfactory than the regular colored papers.

In making the eighteen-inch disks shown in Plates VI. and VII. papers from the Hering series were fastened to thin pieces of cardboard by means of shellac. If glue or mucilage be used, the cardboard bases curl so that they can not be used; but with shellac [1] the disks remain flat without any curling.

At the start eight colors were used (Hering's *R*, *O*, *Y*, *G*, *B*, *P*, *Bk*, *W*). Later the number was reduced to six, since there was considerable friction at the center of the disks. To avoid friction it is recommended that the central hole be about an inch in diameter. For all ordinary work a central hole of an inch should prevent frictional binding.

With these large disks any possible combination of the six, or eight, components may be set up as quickly as the handles can be moved. Also it is a relatively simple matter to shift continuously from one color to any other.

When the disks of Fig. 9 are fastened in pairs it is somewhat of an intelligence test to thread them for actual work. The following instructions for threading the disks may save time and temper. Place the set of *R* disks flat upon a table and then place the *G* disks on top of the *R* as pictured in Fig. 10. Now thread the under surfaces of the *G* disks simultaneously under the upper surfaces of the *R* disks. Having done this place the disks upon the supporting axis, or else hold them firmly at the center with the finger. Holding

[1] This use of shellac was discovered by a mechanician in the laboratory when large disks were being made for another type of differential color wheel.

the handle of the *R* disks, turn *G* counter-clock-wise a complete revolution. These two disks are now ready for use. Any desired combination of *R* and *G* can be obtained by shifting the handles.

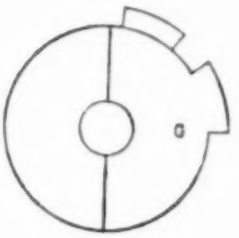

FIG. 10

To add the *B* disks place *B* on top of the *G* and thread the under surfaces beneath the upper surfaces of both *G* and *R*. Turn counter-clock-wise one revolution as above. Any desired combination of *R*, *G*, and *B* can now be obtained by shifting the disks.

Another possible procedure is to place the disks shown in Fig. 9 upon the central axis before the handles have been bound together. This method has the advantage that the thickness of cardboard placed between the handles can be nicely determined before the disks are finally locked.

When all of the disks have been threaded in the proper order a permanent obstruction placed somewhere between the handle of the first and the handle of the last locks the series so that no disk can be removed. Without such an obstruction the disks can always be unthreaded, so that worn disks may be removed, new ones added, and the combination of disks used may be changed *ad libitum*.

(2) Another form of double-sectored disk is mentioned here because, despite its limitation, it may be of value for some purposes. Instructions for making it follow.

Cut out a disk in the form pictured in Fig. 11. Also cut a complete large circle of the same color. The sector and the circle may be regarded as a set. By placing sectors one behind the other it is possible to make the blank 180° any desired combination of the colors. The clear limitation of this form of disk is that there must always be a constant 180° of color or gray in the mixture. With two sets, however, say *R* and *G*, it is possible to get any desired combination

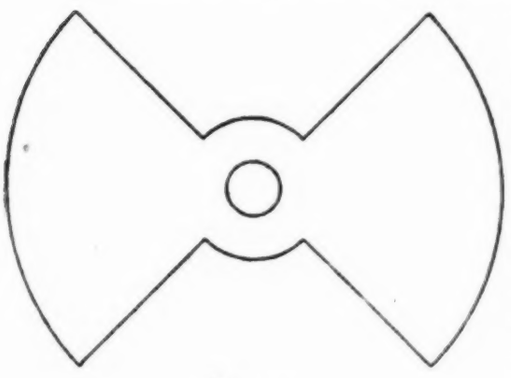

FIG. 11

by placing now the *R* in front and now the *G*, and by shifting the circles. Changes in the neighborhood of 180°*R* + 180°*G*, however, demand a great deal of shifting.

B. Illumination.—It is important to have a uniform illumination upon the color disks. Diffuse light from above is probably the most satisfactory for ordinary work.

When Type I. is used in a dark room two lamps with reflectors may be placed so that one is to the right and the other to the left of the color mixer. This arrangement is not very satisfactory with Type II., *A* (see Plate V.) for the reason that the artificial light is directed into the eyes of the observer. With Type II., *B* (see Plate VI.), however, screens may be placed so that they will prevent direct light from reaching the eyes.

VI. Incidental Results

The chief aim of the present study has been to develop a type of apparatus which may be of value for psychological work. During the course of the study, however, several incidental observations were made, and a note upon these results may be in order.

A. The Demonstration of Contrast.—Beautiful simultaneous contrast colors may be observed with Type II. of the apparatus. To get the contrast colors take a large neutral gray screen and in the center cut a circular hole 5 to 10 cm. in diameter. Behind the hole place a colored paper, say blue, and a cardboard back. Now hold this circle of blue in front of the rotating mirror and slightly below. An 'elliptical' frame of poorly saturated blue will be seen in the mirror image. The frame surrounds a small patch of neutral gray which will clearly take on the contrasting color, yellow. The most satisfactory distance between the mirror and the colored paper depends upon the diameter of the color-patch and upon the angle of the mirror to the axis of the apparatus. The distance is best determined by actual trial. By shifting the colored paper behind the gray screen various contrast colors may be produced.

B. Variation of Chroma.—As the above screen is moved

away from the rotating mirror the chroma of the color frame is gradually reduced, since more and more gray is added to the mixture. By attending to the color frame (seen in the image) and by increasing the distance between the screen and the mirror it is possible to reduce the chroma from maximum to a very low value. For this demonstration the circle should have a diameter of 2 to 3 cm.

C. Visual Purple.—Another interesting observation has been made with Type II. in a dark room. In the wall of the dark room is a window 22 x 25 cm. through which bright sun light is admitted. About 2.5 M. in front of this window and slightly to one side is the rotating mirror facing the window. The mirror image is built up by the rotation of the patch of bright light from the window. When this image is observed it takes on a purple or rose hue. Sometimes the entire field is uniformly colored with this purple and at other times the purple is more sharply limited. In the latter case the boundary of this purple patch is irregular and its shape changes in a manner suggestive of the movements of an amoeba. The observation of these amoeboid movements has been verified repeatedly by the writer and by six other observers. The purple is also observed with a bright artificial light in a dark room; there are sometimes other colors than purple when artificial light is used.

In a preliminary and very tentative way the writer suggests that the purplish color may be the well-known rhodopsin made visible. The alternation between a bright light and total darkness would probably (under certain time and space conditions) bring out upon the retina the well-known purple color. If the retina were covered with a purple substance, it is possible that light transmitted through it would be modified as if there were a purple film immediately in front of the retina. There is no reason why the eye should not project and see its own visual purple under certain conditions. The amoeboid movements of the purple patch definitely suggest some sort of a photochemical change upon the retina. The matter demands further investigation.

D. Flicker.—When the apparatus (both types) is rotating

slowly flicker may be observed. The flicker, for some reason unknown to the writer, gives one the impression of rotation in a direction opposed to that of the apparatus.

One advantage of double-sectored disks is that the flicker rate is relatively low when measured in terms of revolutions per second, since there are twice the normal number of alternations per revolution.

E. Continuous Color Series.—If two pieces of colored paper, say red and yellow, be placed side by side in front of the rotating mirror, the mirror image will show a continuous series from red through orange to yellow. The steps of the series are not psychologically equal. There is a relatively abrupt step at the two extremes. These series are of value in demonstration.

F. Geometrical Projections.—The descriptive geometry of the images seen in Type II. is rather complex. The path of a point when viewed as an image has already been discussed in the mathematical section. This path can be made visible by taking a piece of black velvet or black card and by placing upon it a white point. An oval curve will be seen under the conditions and the shape of this curve will vary with the distance of the point from the mirror and with its angular position relative to the axis of rotation. By arranging colored points upon the black velvet a large number of interesting and beautiful patterns can be built up.

The projection of a straight line is a conical figure. If a short strip of white card be placed upon the black velvet, a cone-like figure will be seen standing out clearly in three-dimensional space. The stereoscopic effects are marked. (See Plate VIII.) If a black line be drawn upon a white card and held before the rotating mirror so that the plane of the card is parallel to the mean plane of the mirror, this line will appear as two parallel lines, with a grayish film between them. If several lines be drawn so that they intersect at a point, the point of intersection vanishes in the mirror image since each line becomes two parallel lines with a grayish film between them.

By combining points and lines, and by using irregular

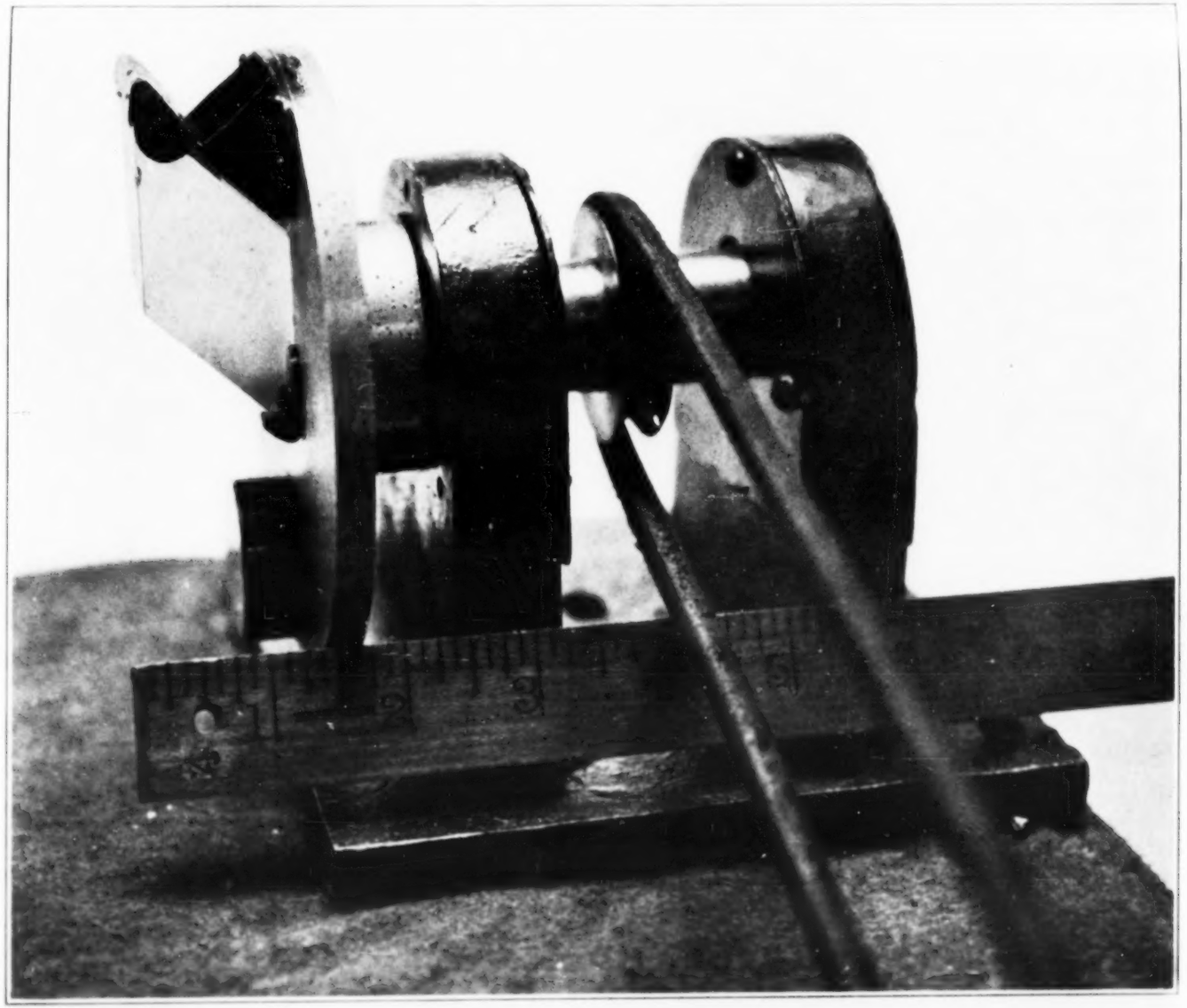

PLATE I

PLATE II

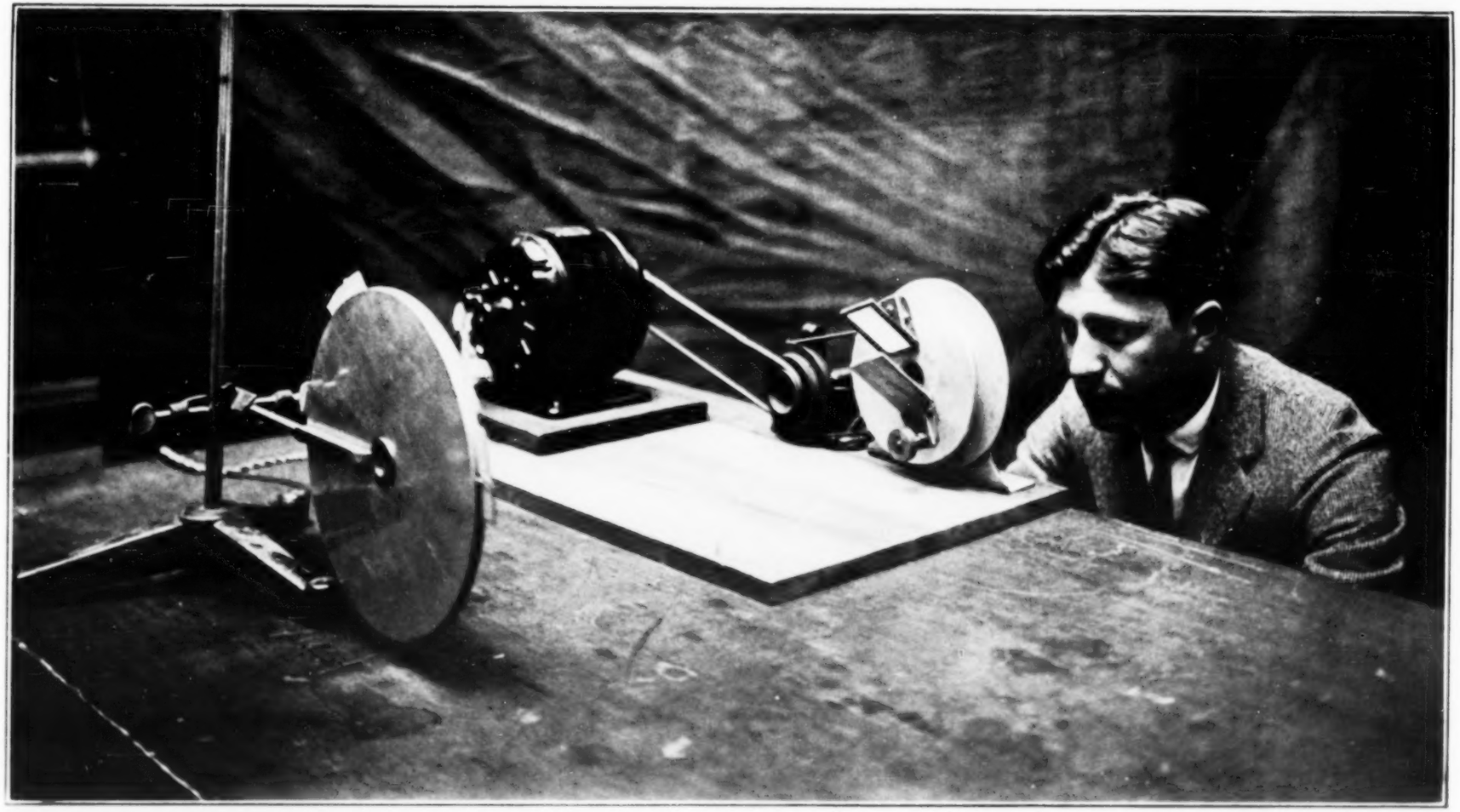

PLATE III

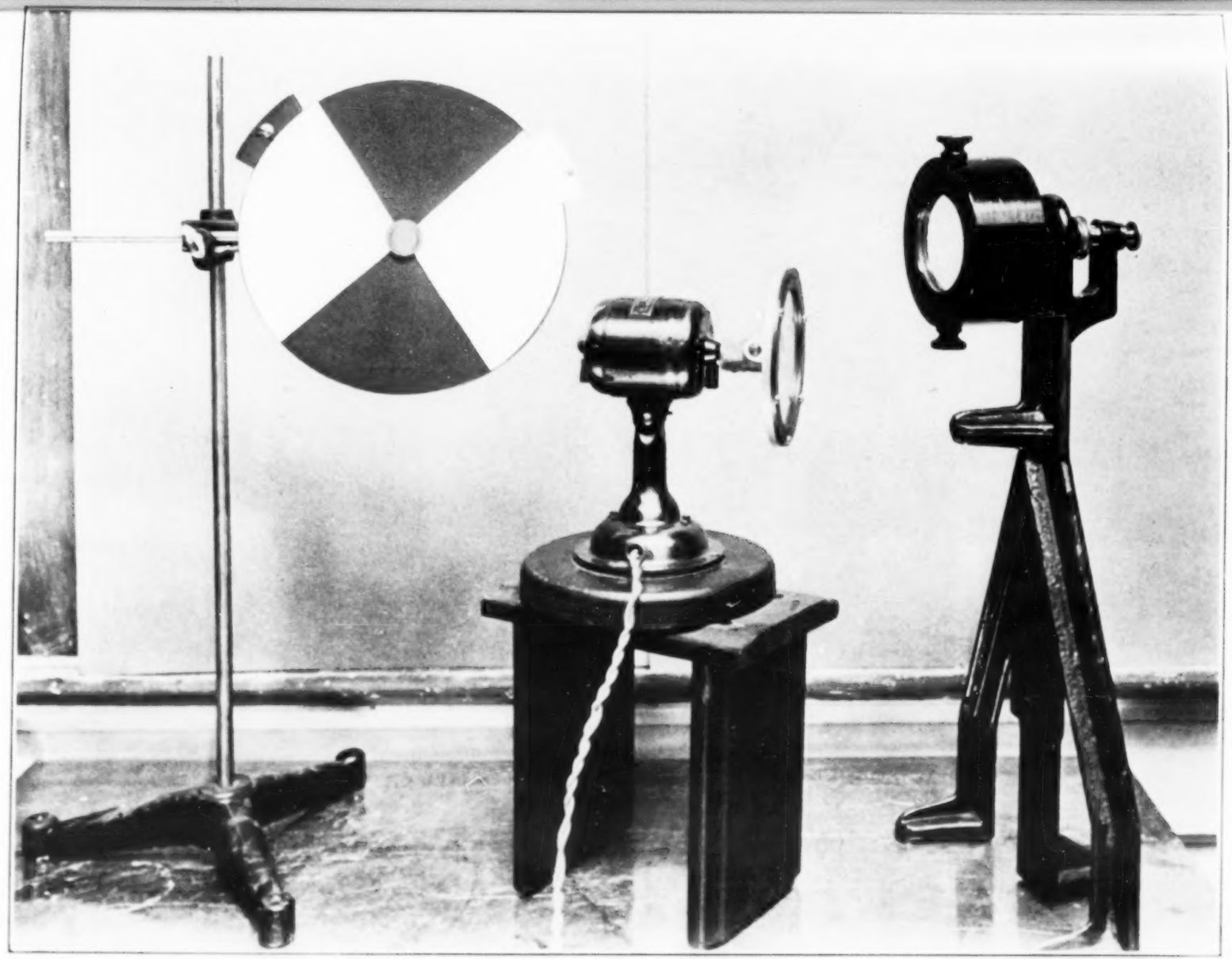

PLATE IV

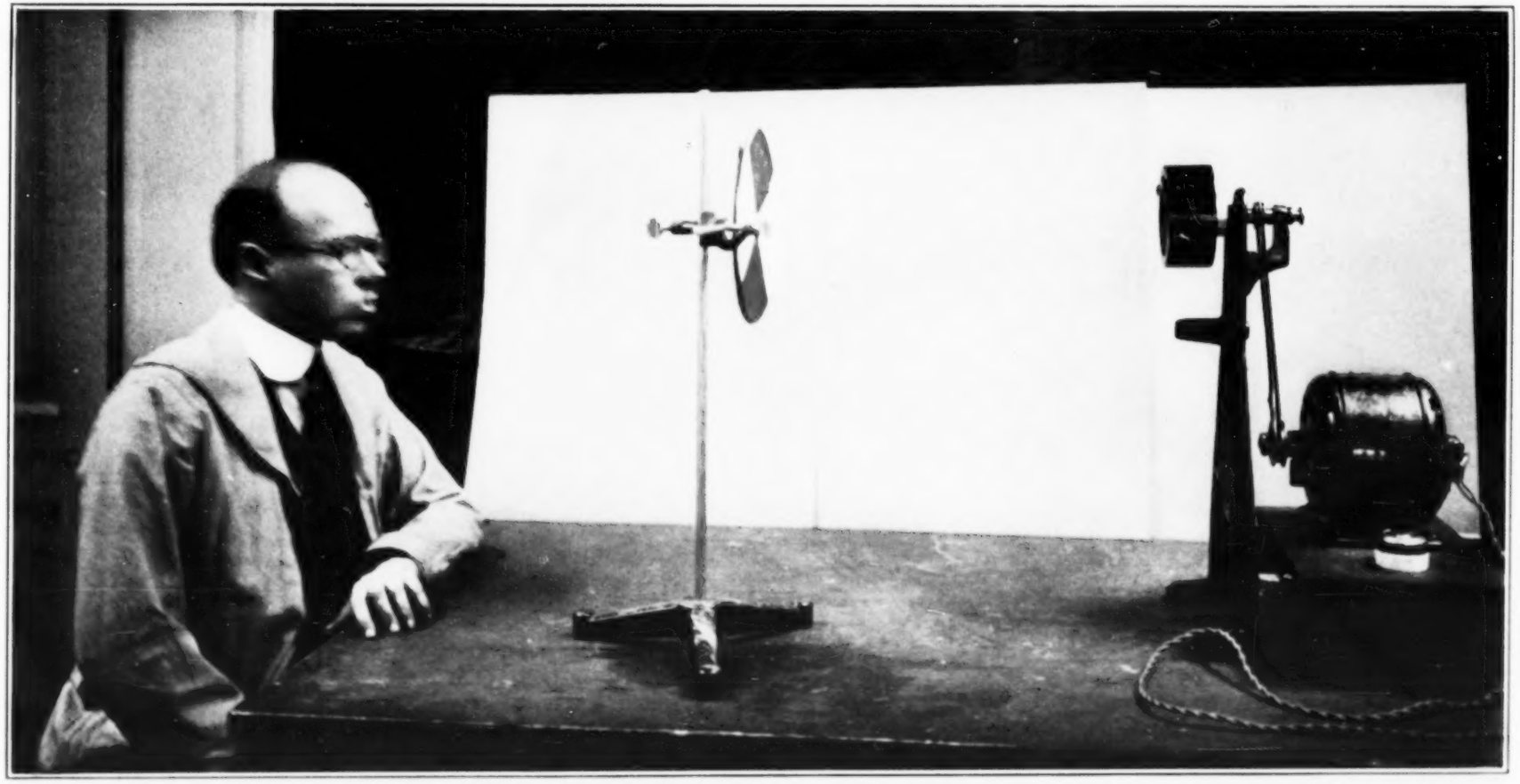

PLATE V

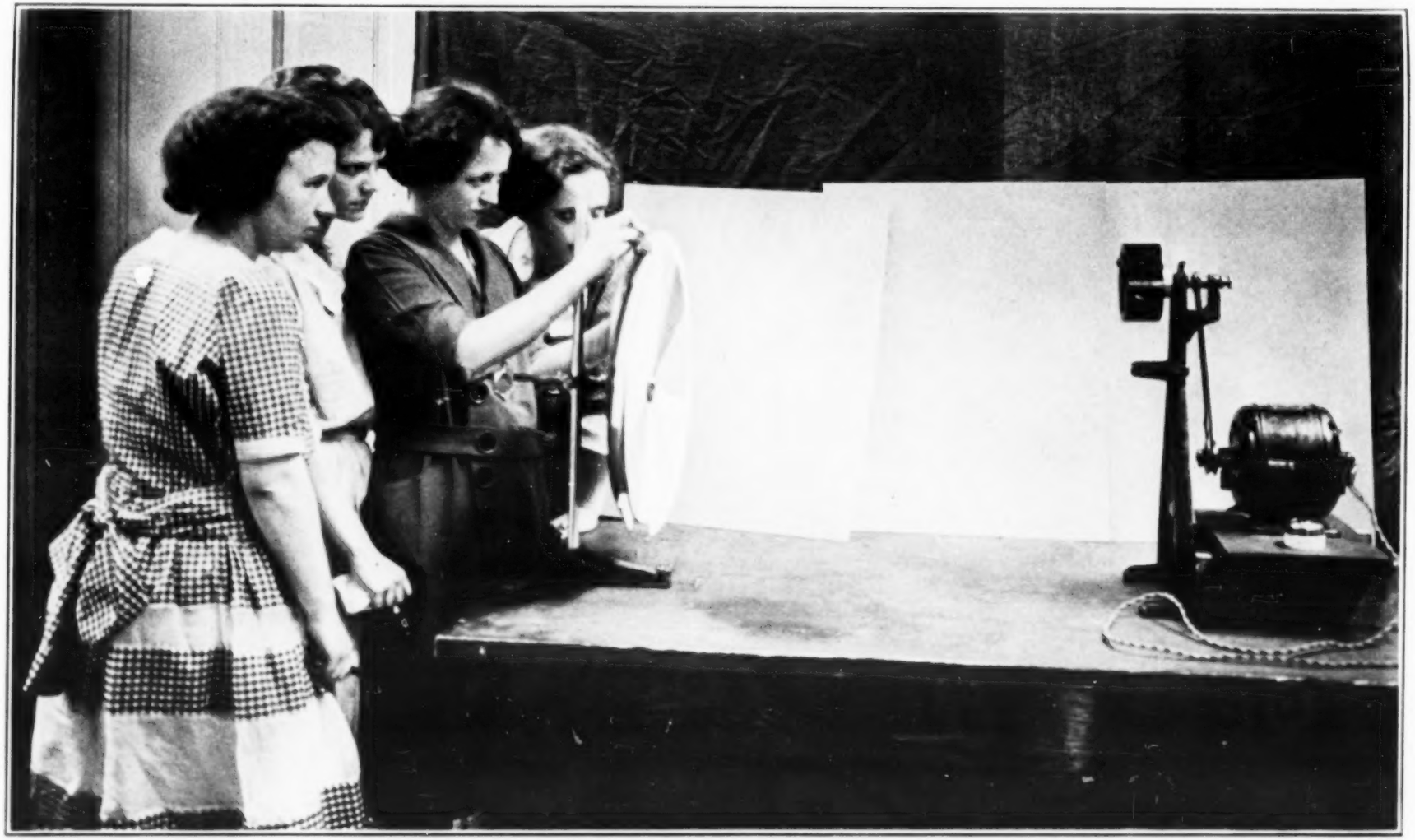

PLATE VI

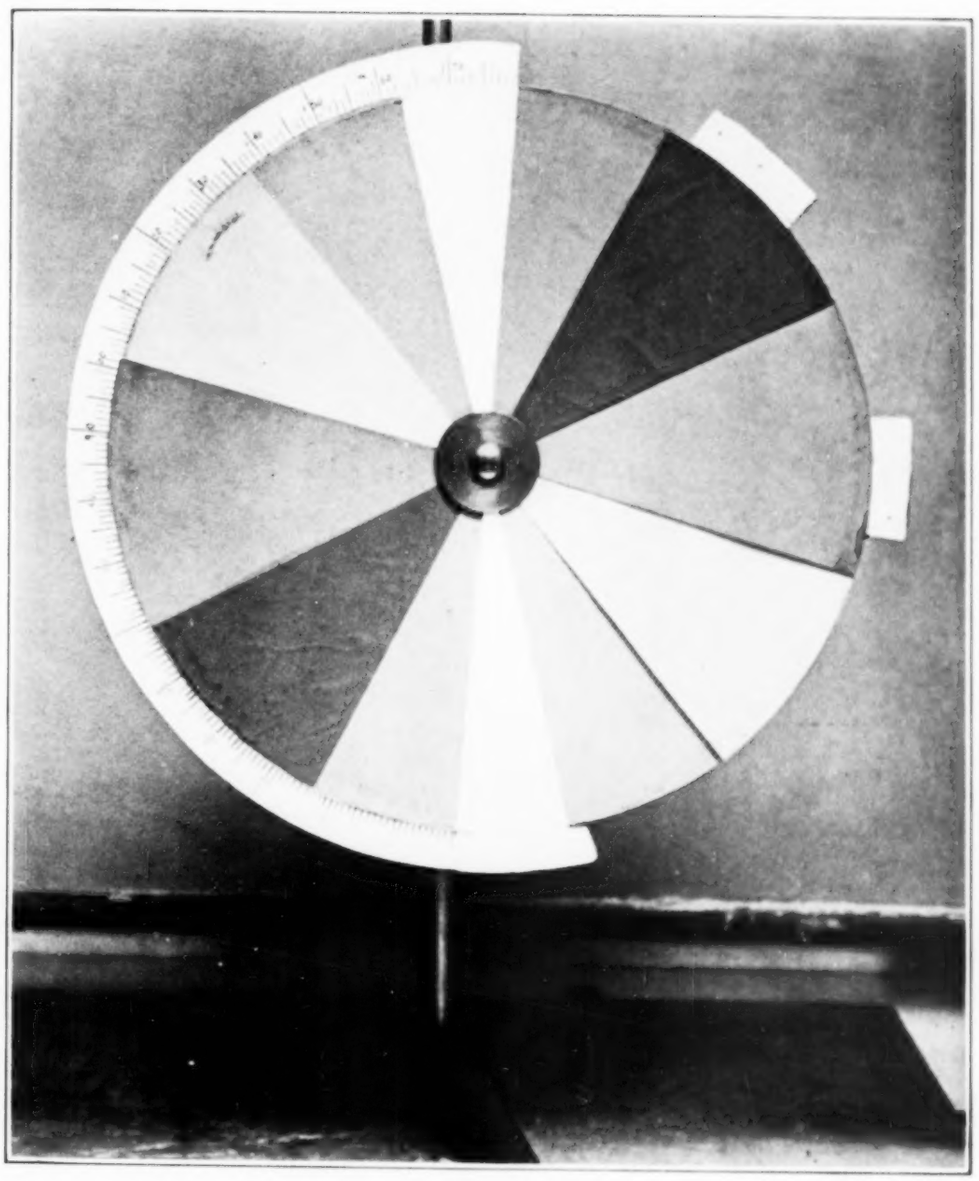

PLATE VII

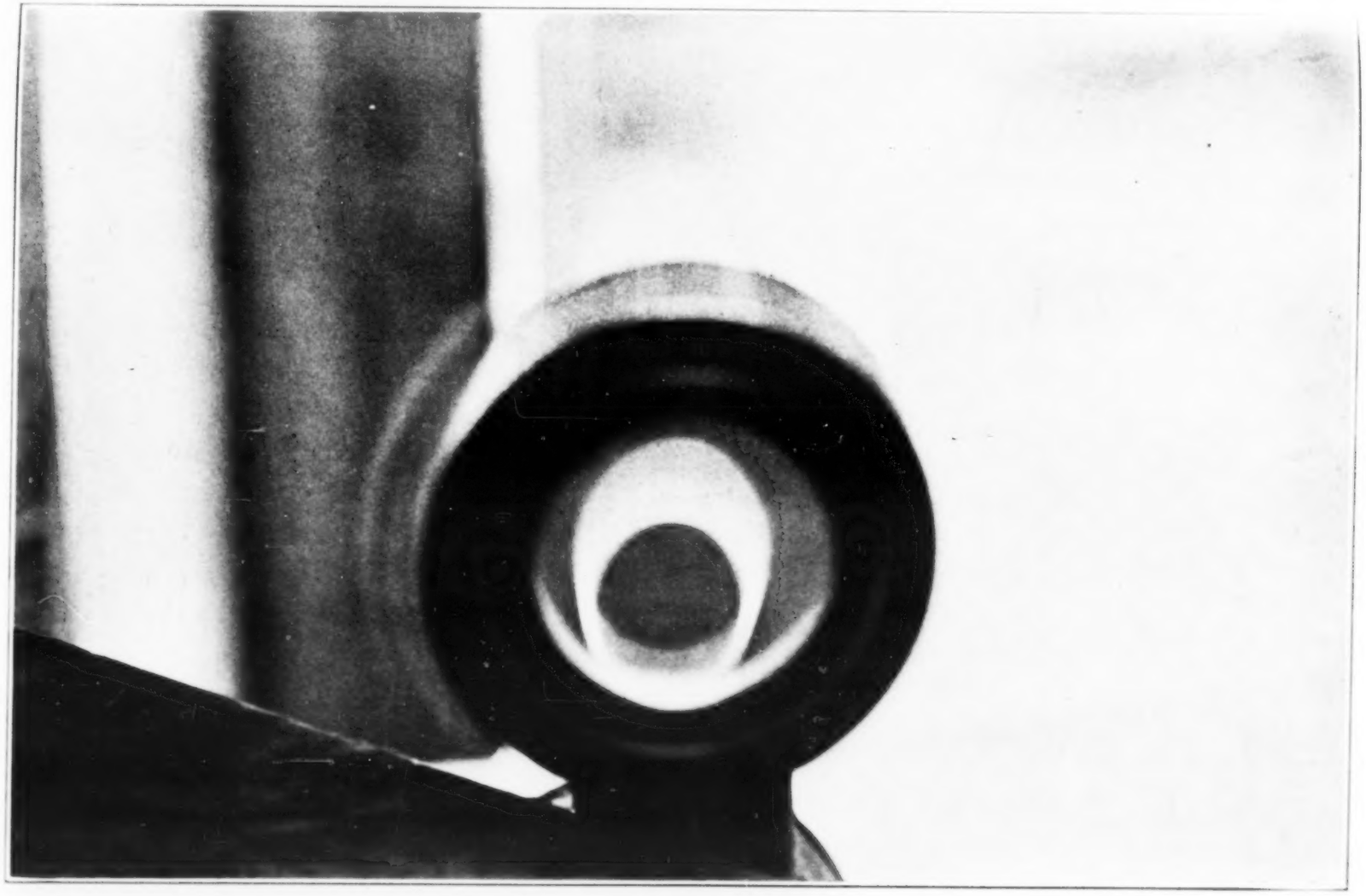

PLATE VIII

surfaces, and by working with various colors, all sorts of interesting mirror images may be developed. If a rotating Maxwell color wheel be viewed in the rotating mirror, many curious changes may be observed as the two rates of rotation are varied.

These latter observations are entertaining and possibly the image constructions may have some æsthetic value.

Summary

The light reflected from stationary color disks may be mixed by means of rotating prisms and mirrors. Several types of apparatus have been tested and two, especially, have proved satisfactory: (1) A rhomb rotated between the eye and the disks about an axis from the eye to the center of the disks. (2) A single plane mirror set at an angle of about 86° to the axis of rotation and arranged so that the mirror image of the stationary disks may be seen. The first type is recommended as a differential color mixer. The second type is useful for certain demonstrations; it may also be used as a color mixer.

Color surfaces for use with the apparatus must be arranged in double sectors in order to give a uniform color field. The construction of special double-sectored disks makes it possible to shift continuously from one color to another without stopping the apparatus.

CORRELATIONS OF INITIAL AND FINAL CAPACITIES IN LEARNING

BY GILES MURREL RUCH

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This paper reports two separate investigations of the correlation of initial and final capacities in learning. The methods of the two studies will be described separately.

I. Improvement in Army Alpha by Adult Subjects

Since there is so little available material for the study of the effects of practice which exists in large numbers of units of work of equal difficulties, one of the most convenient types of practice material is that furnished by the standardized tests of mental ability. Army Alpha has been used for this reason and possesses the double value of existing in five forms as well as providing a rather wide variety of mental functions. The present experiments used five of the separate tests of Army Alpha, as follows:

Test 2. Arithmetic Problems	Time limit	2	min.
Test 4. Synonym-Antonym	"	"	$\frac{3}{4}$	"
Test 5. Disarranged Sentences	"	"	1	"
Test 6. Number Series Completion	"	"	$1\frac{1}{2}$	"
Test 7. Analogies	"	"	$1\frac{1}{2}$	"
		Total	$6\frac{3}{4}$	"

The subjects used were 30 college students, chiefly seniors and graduates in psychology. This group represents the pooling of two experiments under identical conditions, the first experiment being carried out at Stanford University during the summer quarter of 1921, and the second at the University of Iowa during the second semester of the year 1922–23. The practices were carried on as group experiments and the subjects were not informed of their changes in efficiency from day to day. The subjects could, of course, form rough estimates of their progress by noting the increases in the numbers of items completed day by day.

The Alpha tests were administered as follows:

(*a*) Form 5 given under standard conditions of Army manual.
(*b*) Practice series:

Day	1,	Form	6,	5	tests	only,	reduced	time	limits (Initial Ability)
"	2,	"	7,	"	"	"	"	"	"
"	3,	"	8,	"	"	"	"	"	"
"	4,	"	9,	"	"	"	"	"	"
"	5,	"	5,	"	"	"	"	"	"
"	6,	"	6,	"	"	"	"	"	"
"	7,	"	7,	"	"	"	"	"	"
"	8,	"	8,	"	"	"	"	"	"
"	9,	"	9,	"	"	"	"	"	"
"	10,	"	5,	"	"	"	"	"	" (Final Ability)

In the discussions of the experiments, the scores on Day 1 are consistently designated as *initial ability* and those of Day 10 as *final ability* (although the limits of improvement had not been reached). The practices were held on consecutive Tuesdays, Thursdays, and Saturdays. The reduced time limits were found to be stringent enough so that very few workers finished ahead of time and then only toward the close of the experiment.

Table I. presents the correlation between initial and final abilities in each of the 5 Alpha tests and for the sum of the 5 tests. The range of talent involved in the practicing groups is represented by the following data on the scores earned in Alpha under standard conditions:

Mean	158.8
Standard Deviation	27.8
Range	90–203

TABLE I.

CORRELATIONS OF INITIAL AND FINAL ABILITIES IN ALPHA TESTS

Test	r	P.E.
2 Arithmetic Problems	0.556	.085
4 Synonym-Antonym	0.675	.067
5 Mixed Sentences	0.700	.063
6 Number Completion	0.865	.031
7 Analogies	0.803	.044
Sum of 5 tests	0.813	.043

It will be noted that there are marked differences in the

magnitudes of these coefficients of correlation. These differences might be due to:

(1) Irregularities due to adaptations to new test situations. These were probably obviated in part by the regular examination which had preceded.

(2) Differences due to changes in the mental functions involved resulting from possible differences in the rates at which the several tests underwent automatization. This factor of change in mental function under automatization has never been sufficiently recognized in the experimentation on learning in the writer's opinion. Hollingworth repeated the same list of opposites 175 times with a resulting correlation of − 0.08 for initial and final abilities.[1] The mental demands of the first and 175th work units must have been very different. Hollingworth recognized this fact himself, it should be noted.

(3) Differences in the amount of influence of formal education and the amounts of previous practice on the various tests. Arithmetic is obviously greatly influenced by this factor, but many of the others are probably not.

(4) Real differences in the improvability of the several kinds of mental functions.

(5) Differences in the total amount of practice in each function, arising from the fact that the daily time for practice varied from $\frac{3}{4}$ of a minute to 2 minutes.

(6) Effects of unreliability of the measures. As suggested, the time limits were not the same and the types of responses are subject in unequal degrees to chance factors. This last is important for Mixed Sentences (true-false type) but would be insignificant in Arithmetic Problems or Number Series Completion.

No attempt will be made to isolate and study these several factors here except in the case of the last-mentioned. The question of unreliability in the statistical sense will be given detailed attention in a later section.

[1] Hollingworth, H. L., 'Correlations of Abilities as Affected by Practice,' *Jour. Educ. Psych.*, 1913, 4, 405–414.

II. Improvement of Children in Card-sorting, Code-substitution, and Abstract Mathematical Problems

These three tests will be very briefly described here. In each case the subjects practiced for at least 10 days. As before, initial ability refers to scores earned on the first day of practice and final capacity to those earned on the tenth day. The numbers of subjects were as follows: card-sorting 49, code-substitution 58, and abstract mathematical problems 63. The subjects were all children and the significant facts about the range of talent in the groups are given in Table II.

Table II

Measures of Central Tendencies and Variabilities of the Chronological and Binet Mental Ages

Test	Chronological Ages		Binet Mental Ages	
	Mean Yrs.—Mos.	S. D. Mos.	Mean Yrs.—Mos.	S. D. Mos.
Card-sorting	13—8	13.7	13—9	26.9
Code-substitution	14—1	15.0	14—4	34.9
Abstract Mathematical Problems	13—7	7.0	13—9	33.5

In the card-sorting series, the 49 subjects sorted a pack of 100 cards bearing novel designs into a case of 10 compartments, each compartment bearing a label for one of the ten different suits of cards. There were 10 cards of each suit in the pack. Five trials were given daily, or a total of 50 trials for the ten days. The 5 daily trials were averaged to form each day's record. The subjects worked individually under practically laboratory conditions. The time was taken to the nearest fifth of a second. No penalties were inflicted for errors, which were never very numerous.

In the code-substitution test, the 58 subjects worked 20 minutes daily for ten days. Ten minutes daily were devoted to translating from English text into code and ten minutes daily to translation from code into English equivalents. The code test used was the old Civil War code of Healy; later

used in the Binet scale. The key to the code was kept before the subjects at all times, or until voluntarily surrendered by the subjects. The text materials were chapters from Oliver Twist, one chapter to be turned into code, the other being previously written out in the code symbols for translation into English. The daily records were taken in terms of the numbers of letters correctly transcribed in 10 minutes. The subjects started the tests individually and were then amalgamated into groups of six or eight for purposes of group competition. The subjects were not specifically informed of their daily gains except as they could infer these facts for themselves. This is also true of the other two tests used with children.

The abstract mathematical problems test was administered in a similar manner to the above with respect to working conditions. A key card was furnished each subject with the following instructions (greatly abbreviated here):

C is larger than *A*,
A is larger than *B*,
B is larger than *D*.

The task was to think out the solutions to problems like the following:

± *A* plus *B* is equal to *C*,
\+ *C* is larger than *A* minus *B*,
− *B* plus *A* is smaller than *D*.

The subjects were instructed to mark all problems which are invariably true with a plus sign, all those invariably untrue with a minus sign, and all those which may or may not be true depending upon the unknown magnitudes which might be assigned to the letters with a plus-and-minus sign. Three sets of 100 problems were provided and these were repeated in order in cycles of 3 days. The subjects were allowed 15 minutes daily practice, the score being the number of problems correctly solved (corrected for chance by the usual methods). In case a subject finished in less than 15 minutes the entire set of 100 problems, he was given credit for the nmuber of problems he would have solved working at that rate for the full 15 minutes.

Table III. gives the correlations between the initial and final abilities in the three tests used with the children as subjects.

TABLE III

CORRELATIONS OF INITIAL AND FINAL ABILITIES IN CARD-SORTING, CODE-SUBSTITUTION, AND ABSTRACT MATHEMATICAL PROBLEMS

Test	r	P.E.
Card-sorting	0.516	.071
Code to English	0.606	.056
English to Code	0.779	.035
Abstract Mathematical Problems	0.782	.033

III. NOTES ON THE USE OF COEFFICIENTS OF CORRELATION CORRECTED FOR ATTENUATION DUE TO ERRORS OF MEASUREMENT

A list of several possible factors making for differences in the correlation of initial and final capacities has already been given. It is obviously impossible to treat these new data analytically in terms of all of these factors. Two at least need introspective analysis and all argue for detailed controls and experimental study of each variable in succession with the remaining ones held constant. There is one of the list, however, which can be rendered approximately constant within limits of experimental error, by the use of statistical methods, viz., the factor of unreliability and errors of measurement. In all, 9 types of mental functions are involved in the two studies (mental functions being roughly defined in terms of the test situations). That the measures of efficiency in these different tests will vary greatly among themselves in the degree of reliability in the technical sense is evident from the fact that the measures are based upon daily practice periods varying from 45 seconds (Synonym-Antonym test) to 15 minutes (Abstract Mathematical Problems test). As was suggested before these differences in reliability have never been adequately allowed for in most studies of learning. The writer has several times attempted to summarize and compare closely the literature on correlations of initial and final abilities under practice but has found the task impossible due to the great differences in the experimental methods which are reflected in the correlations reported.

It is now well recognized by statisticians that coefficients of correlation are meaningless unless a large number of facts are weighed in the interpretation of such coefficients. These are in part questions of sampling, range of talent, unreliability, curvilinearity, etc. As a control of the factor of dilution of raw coefficients of correlation due to errors of measurement (unreliability), the method of corrections for attenuation has been advocated, notably by Spearman. Several formulas have appeared. These are not quite identical in form but are alike in their basic assumptions, viz.,

(1) Unreliability of measures can be computed by correlating two or more *similar* forms of the test, *i.e.*, by taking two independent sets of measurements of each variable. The concept of similar forms involves complete similarity of the two tests without duplication of identical items. This condition may often be approximately satisfied by drawing random samplings from a large mass of homogeneous test items.
(2) The errors in the two independent sets of measurements are uncorrelated.

If these conditions do actually obtain, the use of the formulas for correction of attenuation due to errors of measurement can be defended. The burden of proof that these conditions have been met must be placed upon the experimentor.

If the raw coefficients are corrected for attenuation, the correlations between initial and final capacities are here (as always) increased in magnitude. In order to bring together some empirical data for comparative purposes, these calculations will be made in several ways. These methods can best be described in terms of the fomulas given below. The notations are consistently as follows:

x_1 First set of measures of initial ability (*i.e.*, scores for day 1 of the practice series);
x_2 Second set of measures of initial ability (*i.e.*, scores for day 2 of the practice series);
x_3 First set of measures of final ability (*i.e.*, scores for day 9 of the practice series);

x_4 Second set of measures of final ability (*i.e.*, scores for day 10 of the practice series);

r_{∞} Corrected coefficient of correlation for initial and final ability.

Four formulas for correcting for attenuation have been rather generally advocated or used in the literature:

Formula I. Advocated by Yule, and defined by the formula

$$r_{\infty\infty} = \frac{\sqrt[4]{r_{x_1y_1}r_{x_1y_2}r_{x_2y_1}r_{x_2y_2}}}{\sqrt{r_{x_1x_2}r_{y_1y_2}}}.$$

Formula II. Used by Spearman in his earlier work, and defined by the formula

$$r_{\infty\infty} = \frac{(r_{x_1y_1} + r_{x_1y_2} + r_{x_2y_1} + r_{x_2y_2})/4}{\sqrt{r_{x_1x_2}r_{y_1y_2}}}.$$

Formula III. Later recommended by Spearman, and defined by the formula

$$r_{\infty\infty} = \frac{r_{(x_1+x_2)(y_1+y_2)}\sqrt{1 + r_{x_1x_2}}\sqrt{1 + r_{y_1y_2}}}{2\sqrt{r_{x_1x_2}}\sqrt{r_{y_1y_2}}}.$$

Formulas IV.a and IV.b. Formula IV.*a* has often been used as an approximate method involving less mathematical computation. Formula IV.*b* has not ordinarily been used but is an analogue of IV.*a*.

IV.*a* IV.*b*

$$r_{\infty\infty} = \frac{\sqrt{r_{x_1y_2}r_{x_2y_1}}}{\sqrt{r_{x_1x_2}r_{y_1y_2}}}, \qquad r_{\infty\infty} = \frac{\sqrt{r_{x_1y_1}r_{x_2y_2}}}{\sqrt{r_{x_1x_2}r_{y_1y_2}}}.$$

It will be noted that formulas I. and II. involve the calculation of 6 raw coefficients of correlation, formula III. involves but 4 raw coefficients, and formulas IV.*a* and IV.*b* involve 4 raw coefficients. Formulas I., II., and III. involve all four of the sets of observations making up the experimental data but this is not the case with IV.*a* and IV.*b* which use but two each of the four possible intercorrelations in their numerators. The denominators are identical or analogous throughout.

At the risk of too great digression from the main task the writer has ventured to include the corrected coefficients obtained by the use of each of these formulas for the sake of rough empirical checks upon the merits of each. Since the raw coefficients entering into each calculation are the same, the comparisons should possess some degree of validity. Table 4 gives the raw coefficients for all data, the subscripts being explained above.

Table IV

Raw Coefficients of Correlation for All Tests

Test	$r_{x_1x_2}$	$r_{y_1y_2}$	$r_{x_1y_1}$	$r_{x_1y_2}$	$r_{x_2y_1}$	$r_{x_2y_2}$	$r_{(x_1+x_2)(y_1+y_2)}$
Alpha:							
2. Arith. Prob.	.769	.869	.757	.556[1]	.851	.715	.793
4. Syn.-Ant.	.713	.777	.779	.675	.667	.763	.826
5. Mix. Sent.	.664	.619	.503	.700	.687	.815	.786
6. Num. Comp.	.851	.916	.773	.865	.767	.772	.821
7. Analogies	.845	.746	.730	.803	.744	.756	.884
Sum of 5 tests	.944	.897	.858	.813	.923	.949	.907
Card-sorting	.718	.877	.550	.516	.843	.790	.719
Code to English	.809	.966	.635	.606	.761	.747	.750
English to code	.848	.882	.789	.779	.769	.782	.806
Abst. Math. Prob.	.846	.981	.811	.782	.845	.845	.885

Table V. gives the corrected coefficients calculated from the raw values in Table IV.

Table V

Coefficients Corrected for Attenuation

	Formula				
	I.	II.	III.	IV*a*.	IV*b*.
Alpha:					
2. Arith. Prob.	.870	.880	.882	.841	.900
4. Syn.-Ant.	.966	.969	.968	.901	1.036
5. Mix. Sent.	1.039	1.055	1.006	1.082	.999
6. Num. Comp.	.898	.900	.876	.923	.875
7. Analogies	.954	.955	.999	.974	.936
Sum of 5 tests	.961	.963	.946	.941	.981
Card-sorting	.828	.847	.814	.826	.831
Code to English	.774	.777	.801	.768	.779
English to code	.902	.902	.869	.895	.908
Abst. Math. Prob.	.900	.901	.925	.892	.909

[1] The values in this column are those reported in Tables I. and III., and are the correlations of initial and final abilities as previously defined.

Probable errors for most of these values can be computed by means of formulas derived by Truman L. Kelley. The earlier formulas for the probable errors of corrected coefficients which were used by Spearman give much higher values for the probable errors than are yielded by those advocated by Kelley who has criticized the older formulas.[1] Kelley recommends the following formula for use with the Yule formula (Formula I. above):

$$\text{P.E.}_{r_{\infty\infty}} = .6745\,\frac{r_{\infty\infty}}{2\sqrt{N}}\left(4r_{\infty\infty}^{2} + \frac{1}{r_{\infty\infty}^{2}} + \frac{1 + r_{x_1x_2} + r_{y_1y_2}}{r^2} + \frac{1}{r_{x_1x_2}^{2}} + \frac{1}{r_{y_1y_2}^{2}} - \frac{4}{r_{x_1x_2}} - \frac{4}{r_{y_1y_2}} - 2\right)^{\frac{1}{2}}.$$

Here,

$$r = \frac{r_{x_1y_1} + r_{x_1y_2} + r_{x_2y_1} + r_{x_2y_2}}{4}.$$

Since the Yule formula for correction for attenuation is a standard form and probably more valid than any other with the possible exception of Spearman's latest recommendation (formula III.), the main discussion of present results will be based upon Yule's formula and the probable errors will be stated for this formula only. Table VI. gives these facts.

TABLE VI

Test	$r_{\infty\infty}$	P.E.$_{r_{\infty\infty}}$
Alpha:		
2. Arith. Prob.	.870	.031
4. Syn.-Ant.	.966	.040
5. Mixed Sent.	1.039	.049
6. Num. Comp.	.898	.033
7. Analogies	.954	.035
Sum of 5 tests	.961	.015
Card-sorting	.828	.042
Code to English	.774	.041
English to Code	.902	.025
Abst. Math. Prob	.900	.021

Inspection of Table V. shows that there is a very substantial agreement among the corrected coefficients yielded by formulas I., II., and III. Equations IV.*a* and IV.*b* are not so satisfactory in their correspondence to the other three

[1] Kelley, T. L., 'Statistical Method,' 1923, Macm., 208–212. The subscripts have been changed to fit the previous notations.

methods. Although ten sets of coefficients are very meager data for generalization, certain of these methods show rather definite promise of superiority over the others. Formulas I., II., and III. do take into account all of the measures entering into the experimental data, a fact which does not hold for formulas IV.*a* and IV.*b* which neglect two of the intercorrelations which might stablize the numerator of the expression. The only excuse for the use of these last two formulas, then, would be the saving of labor in the calculations. This might be justified for rough practical work were it not for the fact that formula III. is probably even less involved and seems to agree closely enough with the more laborious formula of Yule. Since the main purpose of calculating corrected coefficients is that of answering certain theoretical questions, it would seem that it is inadvisable to sacrifice such ends for practical reasons. Formula II. would seem to be as good as I. or III. It differs from I. only in the use of the arithmetic mean instead of the geometric mean in the numerator. In the writer's opinion, Yule's method should be used unless practical considerations prohibit the amount of labor, in which case Spearman's recommended formula might be substituted. Formulas IV.*a* and IV.*b* would therefore not be very valuable.

IV. Discussion and Summary of Results

Accepting the corrected coefficients as probable estimates of the truth (within the limits of the probable errors and fundamental assumptions of the method), inspection of Table 6 and elsewhere forces the conclusion that there are important differences in the initial and final efficiencies in the various mental functions involved in the present studies. The range of the corrected coefficients is from .774 ± .041 (Code to English) to 1.039 ± .049 (Mixed Sentences). The latter value should be considered as 1.000 or less. As is known to statisticians, the real difference between a correlation coefficient of 1.000 and .774 is very much greater than appears to be the case in terms of the subtractive difference of these values. Coefficients of correlation are best interpreted in

terms of a scale of magnitudes based upon the closeness of prediction possible for various values of r. Such a scale of real values is furnished by the *standard error of estimate.* The formula for the standard error of estimate of a second variable knowing a first variable is $\sigma_{2\cdot1} = \sigma_2\sqrt{1 - r_{12}^2}$. Kelley has used this principle in even a more meaningful form in what he calls the 'Coefficient of Alienation,' [1] thus:

$$k = \sqrt{1 - r^2}.$$

Solving for the k values gives us the extent to which prediction falls short of unity and hence values of r can be compared with a numerical meaning quite lacking in the r values themselves. Applying the k formula to the values in Table 6, we get the following results.

TABLE VII

ALIENATION OF THE INITIAL AND FINAL ABILITIES ($k = \sqrt{1 - r^2}$)

Test	r	k
Alpha:		
2. Arith. Prob.	.870	.493
4. Syn.-Ant.	.966	.258
5. Mix. Sent.	1.039 (1.000)	.000
6. Num. Comp.	.898	.440
7. Analogies	.954	.300
Sum of 5 tests	.961	.277
Card-sorting	.828	.561
Code to English	.774	.633
English to code	.902	.432
Abst. Math. Prob.	.900	.436

In accepting these k values, it should be born in mind that they are based upon corrected coefficients and not those experimentally obtained. *Corrected coefficients are at best promises of truth, not demonstrated truth.* Further, it must not be forgotten that we cannot predict scores on day 10 from those on day 1 with as small errors as the k or $\sigma_{1\cdot2}$ values would indicate using the $r_{\infty\infty}$ coefficients. Corrected coefficients cannot be used in regression equations for purposes of estimate. The raw coefficients ($r_{x_1y_2}$) of Tables I., III., and IV. must be used for this purpose. The k values for the raw coefficients are very much larger. The k values of Table

[1] Kelley, *op. cit.*, p. 173.

VII. do show, however, that there were great differences in the correspondences of scores earned in initial trials with those earned in the final trials, function by function. In some cases, *e.g.*, Mixed Sentences, the relationship is very close according to our estimates, in others, *e.g.*, code to English, it is very imperfect. Some of these differences can be explained upon the basis of the total amounts of practice involved in the different tests. Those from Army Alpha provided much less true practice since the mental functions were exercised but from $7\frac{1}{2}$ to 20 minutes during the entire experiment. The tests used with the children provided as much as 100 to 150 minutes of exercise. Even at that the prediction is about as close for 100 minutes of English to code or 150 minutes of Abstract Mathematical Problems as for 20 minutes of Arithmetic Problems or 15 minutes of Number Series Completion. The conclusion from these considerations would seem to be that there are real differences in the correlations of initial and final efficiencies which are over and above those which can be explained in terms of the amounts of intervening exercise in terms of time units. Even if the validity of the corrections for attenuation is challenged, this conclusion can still rest on the evidence of the raw coefficients. It is still possible that the differences found among the various tests might be accounted for in terms of differences in the ranges of talent employed. That this cannot account for all, or even much, of the differences is evident from the fact that the same range of talent was involved in all the Army Alpha tests and that the talent was practically the same in both code-substitution tests and the Abstract Mathematical Tests. It is true that there is no way of equating the talent involved in the two separate experiments, *i.e.*, the one using adults and the one using children.

It is hoped, in conclusion, that these data will serve to emphasize the complexity of the problem discussed, even if, admittedly, the present results must be considered as very inadequate for the settling of any very fundamental issues.

CRITIQUE OF THE ASSOCIATION TEST AS APPLIED TO ADVERTISING

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I. Introduction

Advertising is one of the first fields to be invaded by the applied psychologist Advertisers in general have come to look upon their psychological contemporaries with no little esteem and expectation. Due, perhaps in a large extent, to the 'self-help' and 'five hundred per cent. mental efficiency increase' advertisements of a pseudo-psychological nature the conscientious psychological worker has an enormous expectancy to live up to.

An easy way to fulfill this high expectancy is to carry over to the field of advertising certain classical experiments that furnish the structuralist one of the chief bulwarks of his stronghold. The association tests and experiments may be carried over bodily to this field of applied psychology, as, indeed, they have been done. Or, as also has been done, the association method may be applied to reactions of advertised brands and commodities.

The question has been raised, however, by practical advertising men as well as by some students of psychology, regarding the validity of transferring the results of these association tests to a situation where it is not how people *think first*, but what they *buy or use*, that tells the tale. It was the purpose of the brief experiment to be reported to attempt an experimental check up on precisely this question.

II. The Experiments

(*a*) *Preliminary Experiment.*—In the fall of 1921 in connection with a course in the psychology of advertising a typical association experiment was given, concerned primarily with forward and backward associations. One list of words,

numbering twenty-five, was given of certain commodities, the association to be a brand. Such words were tires, hosiery, chocolates, shoes, etc. As a by-product of this experiment twenty-three observers were later asked to write down the brand of talcum powder that they were using or purchased if they were consistent in purchasing the same brand from time to time.

(*b*) *Main Experiment on Agreement in Association and Use.*—The preliminary experiment was followed up in the fall of 1922 with the forty-two members of a class in advertising as the subjects. Fortunately the subjects were equally divided between males and females, making comparison in direct numbers easy. The group method was followed. Only responses were recorded, the association time being neglected for the time being. The following list of stimulus words was read singly by the experimenter in the order numbered:

1. shoes
2. chewing gum
3. tooth paste
4. suits
5. hosiery
6. fountain pen
7. watch
8. candy
9. underwear
10. overcoats
11. cigarettes
12. toilet soap
13. breakfast food
14. talcum powder
15. magazine

The subjects were instructed beforehand as to the nature of this phase of the experiment and told to respond with the first brand name that they could think of. Three days later they were given the same list of words under similar conditions except that this time they were to respond with the brand of the commodity that they consistently used. In case they had no favorite or consistently used no particular brand they were to respond with a blank.

(*c*) *The Experiment in Consistency in Associations.*—Four months after *experiment b* was performed 28 members of the original class were again given the list of fifteen words to which they were to respond with the first brand that they

thought of. A comparison of these results with those of the earlier experiment will give insight into the consistency from time to time in thinking brands, and throw additional light onto the general problem under investigation.

III. The Results

(*a*) *The Preliminary Experiment.*—Table I gives the results of the agreement of talcum powder associations with talcum powder used. Under the column headed 'agreement' are recorded the cases in which the powder first thought of was the one used. The 'disagreement' column gives the cases in which the brand used was not the brand first thought of. Under the head of 'irrelevant' are recorded those cases in which the subject did not use a powder, or had no consistent preference, or had been unable to respond with the name of a brand in the associations, due to some complex on the topic of talcs. The per cent. calculations are based upon those cases in which the associations or brand used were strictly relevant. This is true for all the calculations given in this report. Irrelevant associations and no particular brand used are considered as not a part of the data. This means that each per cent. has been calculated largely upon an individual basis.

Table I

Preliminary Findings on Agreement between Restricted Association and Use of Talcum Powder, by Sex, Cases, and Per Cent.

Agreement				Disagreement				Irrelevant	
Male		Female		Male		Female		Male	Female
Cases	%	Cases	%	Cases	%	Cases	%	Cases	Cases
3	50	2	25	3	50	6	75	8	1
Avg. per cent. 37.5				62.5					

These data show practically twice as much disagreement as agreement for the group as a whole. By sexes there is greater disagreement among the females between associations and uses.

(*b*) *The Main Experiment.*—Table II gives the summary

of the data of this experiment, tabulated similar to Table I. Again we find the disagreement greater than the agreement. Again there is greater agreement in the case of the males, it being 61 to 54 in terms of per cent.

Obviously all the items are not to be grouped in the same economic class. In Table III the commodities are arranged into three economic groups, and the cases in agreement only recorded, and in terms of per cent. The disagreements may be calculated mentally by subtracting the figure given from 100.

TABLE II

AGREEMENT, OR DISAGREEMENT, OF ASSOCIATIONS WITH USES BY SEX AND COMMODITY

	Agreement		Disagreement		Irrelevant	
	Men	Women	Men	Women	Men	Women
Shoes	5	3	3	6	13	12
Chewing gum	13	8	5	8	3	5
Tooth paste	19	11	2	10	0	0
Ready made suits	11	1	6	1	4	19
Hosiery	5	9	5	7	11	5
Fountain pen	10	16	11	3	0	2
Watch	12	8	8	3	1	10
Candy	5	12	10	6	6	3
Underwear	7	0	8	0	6	21
Overcoats	2	2	2	1	17	18
Cigarettes	9	0	2	3	10	18
Toilet soap	9	9	6	11	6	1
Breakfast food	12	7	7	8	2	6
Toilet powder	10	10	6	9	5	2
Magazines	11	8	7	11	3	2
Totals	140	104	88	87	87	124

The agreement is thus found to be greatest in the case of the 'other essentials,' next greatest in the 'Luxuries' group, and least in the 'clothing.' In no instance, however, is this agreement great enough to give validity to the association tests as a final test of the measure of an advertising series.

The sex differences are greatest in the 'clothing' group, smaller in the 'other essentials' and least in the 'luxuries.'

Whether or not these are real sex differences is an important question. It may be that they are made apparent merely by the fact that advertising of certain of the commodi-

ties does not make an attempt to appeal directly to both sexes. Cigarettes offer a good example of what is meant; the same applies, but in an opposite direction, in the case of toilet powder.

TABLE III

AGREEMENTS, IN PER CENT., ARRANGED ACCORDING TO ECONOMIC CLASSIFICATION OF COMMODITIES

	Males			Females		
Clothing:						
shoes	62.5			33.3		
suits	64.7			50.0		
overcoats	50.0			66.6		
underwear	46.6			0.0		
hosiery	50.0			56.2		
Avg.		54.7			41.2	
Other essentials:			47.9			
breakfast food	63.2			46.6		
toilet soap	60.0			45.0		
watch	60.0			72.7		
fountain pen	47.6			84.2		
tooth paste	90.4			52.3		
Avg.		64.2			60.1	
Luxuries:			62.1			
chewing gum	72.2			50.0		
candy	33.3			66.6		
cigarettes	81.8			0.0		
toilet powder	62.5			52.6		
magazines	61.1			42.1		
Avg.		58.2			54.3	
			56.2			

Accordingly five stimulus words were selected out of the list whose appeal in advertising the commodity is asexual. These are listed in Table IV.

TABLE IV

COMPARISON OF THE SEXES IN AGREEMENT IN ASEXUALLY ADVERTISED COMMODITIES IN PER CENT.

	Males	Females
Fountain pen	47.6	84.2
Tooth paste	90.4	52.3
Chewing gum	72.2	50.0
Magazines	61.1	42.1
Hosiery	50.0	56.2
Averages	64.3	58.8

In the case of these commodities in which a valid comparison between the sexes may be made it is found that the difference earlier noted still persists. The agreement is greater with the male subjects. This difference may mean that men are more influenced by advertising in their pur-

chases and habits than are women. It may mean simply a tendency toward flight from reality in female associations.

It is quite apparent from these results that the association test is not a valid test of advertising. All other things being equal it might be a valid test; but the other things are not equal. There is many a slip between the ad and the lip. These slips may be caused by economic reasons. Especially is this true in the group of subjects used. In most instances they were squeezing an educational opportunity out of limited funds; most of them had to rub the eagle some time before they parted with him. In this they are probably representative of the great bulk of purchasers.

Habit is another factor that might cause a lack of agreement between strong associational bonds created by certain advertising and the use of the general commodity under consideration. New brands might have better immediate associational value, but the habit of use may have been formed before the thought habit. The results which have been presented indicate that the thought habit does not break up a use habit.

The invalidity of the association test is well shown by a few examples. Deducting from the associations to the list word for 'magazine' one would infer that The American Magazine was the most read by this group of college students. But in actual use it is The Saturday Evening Post. Another example from tooth pastes: Pepsodent was the most thought of, but both Colgate's and Pebeco are used more by this group than Pepsodent. Cigarettes, limiting this to men only, also show up the invalidity of the association test results being lifted bodily into advertising; among the men who smoked cigarettes twice as many thought of Camels as actually smoke this brand.

An attempt was made to ascertain to what extent financial motives might cause the disparity between association and use. For each of the forty-two subjects the per cent. of individual agreement was computed. These range from 22.2 to 92.9. The highest agreement was in the case of a male, the son of a wealthy rancher, who had been accustomed

to having all his needs, real and imagined, gratified. But the 22.2 per cent. agreement was made by a girl, in similar straits so far as ready money is concerned, but unpopular and thought to be queer by her associates.

It is of course difficult to get a direct light onto the financial straits of the subjects. In the particular school fraternity living does not indicate financial flushness. About the only valid basis for separating the students into two groups is on the basis of working their way through school, with no outside aid other than that of the student loan funds.

Table V gives the distribution of agreement, by per cent. intervals, first, by sexes, then the total group, then the portion of the group (twelve) who are dependent upon their own labor and ingenuity for their support while in college.

TABLE V

DISTRIBUTION OF PER CENT. AGREEMENT BY GROUPS

Per Cent. Interval	Males	Females	Total	Self-supporting Students
90–99	1	0	1	0
80–89	2	2	4	1
70–79	3	2	5	1
60–69	7	4	11	2
50–59	4	2	6	3
40–49	2	7	9	2
30–39	2	3	5	3
20–29	0	1	1	0
10–19	0	0	0	0
0–9	0	0	0	0
Totals	21	21	42	12

To enable readier comparison the groups are charted in Chart 1, having in the meanwhile been reduced to a basis of per cent. of the group, rather than a simple frequency distribution. Again the women are below the men in agreement between association and use, while the self-supporting students as a group composed of both sexes fall still lower in agreement. It would thus seem that financial reasons may be at work causing the discrepancy between association and use. The persistent sex differences which have been noted at all parts of the experiment may be due in a large measure

to the virtue of prudence being characteristic of the women enrolled in the class and who acted as subjects.

(*c*) *Consistency in Associations.*— Twenty-eight members of the original group of subjects were given the same stimulus list four months after its first presentation. They were again instructed to write down the first brand that they thought of. The second response to eachstimulus was compared with the first response to the same stimulus word four months previously.

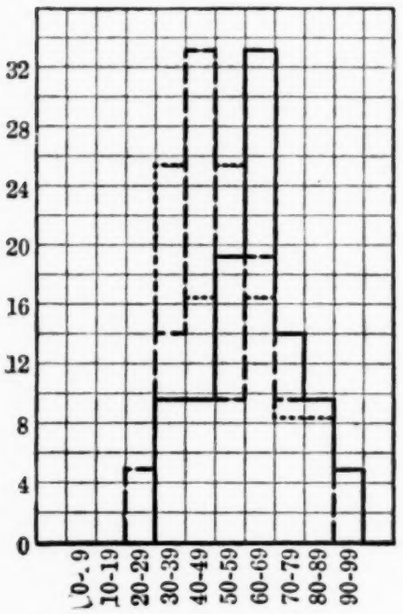

CHART 1. Distribution, in per cent., of individuals with per cent. agreement between association and use indicated along the base line.

Solid line — All men
Broken line — All women
Dotted line — Self-supporting students, both sexes

In the case of the nineteen men the per cent. of agreement (consistency) was 48. In the case of the nine women the per cent. of agreement was 62. This reverses the sex differences which have been noted so far. This consistency of associations after a four-months interval is about as low as the agreement between association and use.

This consistency, or rather lack of it, casts grave doubts upon the validity of the application of association tests in practical situations.

III. SUMMARY AND CONCLUSIONS

Two types of experiment were reported. In the first type a comparison was made between associations to stimulus words of certain commonly used commodities and the brands of these articles actually used by the subjects. The results with forty-two subjects indicate that:

1. There is but little agreement between brand thought of and brand used;

2. This agreement is greatest in essentials other than clothing, next greatest in the luxuries, and least in articles of clothing;

3. Persistent sex differences were noted which seem to indicate that the disparity between brand thought of and brand used is greater in women than men;

4. Financial reasons may make the use different from the associations as indicated by separate data.

In the second type of experiment the individual consistency in restricted association to commodities with the brand was tested after a lapse of four months. This showed that:

5. The lack of agreement in restricted associations of this sort after a four-months interval is about as great as the lack of agreement between association and use;

6. The sex differences here go in a direction opposite to that found in the first type of experiment.

From both types of experiment we must conclude that, while the association tests may be invaluable to the structuralist in ascertaining how the mind is composed, they do not carry over with equal value to a practical situation such as is presented in advertising. There is many a slip between the thought and the purchase, which is really of genuine importance in the commercial field, and which is glossed over and ignored in an association-test psychology of advertising. The association test in this field denies the existence of the very things of paramount importance.

A NEGLECTED PHASE OF THE PART-WHOLE PROBLEM

BY HELEN L. KOCH

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The specific problem we have submitted to investigation is that of the relative value of the whole as opposed to the part method in sensori-motor learning demanding the simultaneous integration of simpler, or so-called 'part,' reactions.

Considerable effort has been expended in an inquiry into the question of the relative efficiency of the whole and part methods of attack in problems in which the reaction to be developed can be broken up into a succession of unit reactions and these units, when the part method is employed, can be mastered separately before a combination of them is attempted. Many excellent researches have been conducted on this type of successive integration in the fields of memorizing and sensori-motor learning. The question of the efficacy of the whole and part methods of learning, furthermore, has not been without its share of attention in those cases in which the reaction to be mastered can be reduced to unit reactions that function simultaneously, but which may be attacked separately before simultaneous integration is essayed. The investigations have occurred largely in the field of perceptual learning. Pedagogues, for instance, long ago discovered that the analytic method of teaching reading was much more cumbersome and unproductive of speedy results than the word or phrase methods.

No carefully controlled work, however, has been directed to the determination of the value of the part method in the acquisition of motor habits when the unit reactions learned first separately are simultaneously functioning parts of the habit to be ultimately developed. This is our problem. The question is an important one for the pedagogy of handwriting, piano-playing, dancing, swimming, etc. Some prac-

tical problems in the field of pedagogy might be stated concretely thus: Is it better in teaching handwriting to let the students practice arm movements first and then drill them on wrist and finger control or to give attention to both of these problems from the start? Is a two-handed attack on a piano composition provocative of more rapidly accruing results than single-handed practice before the two hands attempt to execute a simultaneous performance?

We chose as the medium for the investigation of our problem the task of simultaneous performance on two typewriters. The hands retained a constant position during the experiment. The order in which the fingers pressed the keys was different for each hand. The sequence for the fingers of the left hand was 321542145, the same numbering for the fingers being employed as is usually adhered to in musical notation. The number one indicated the thumb; five, the little finger. The right hand, at the same time, performed in accordance with the sequence, 243215452. It should be noted that all five of the fingers were used on the keys of the instrument, contrary to the usual technique employed in typewriting, in which the thumb is kept on the spacer. The fingers played on the first five keys of the middle row of each machine, the position of the keys being first with reference to the space between the typewriters. The task to be mastered consisted, not only in learning to manage the two hands simultaneously, but to press the keys at a rate of one for each beat of a metronome set at 140.

This particular speed of the metronome was chosen somewhat arbitrarily but not entirely. It was our intention to make the problem sufficiently difficult that the details of the method of adaptation would not be obscured by too rapid learning. It must be remembered, then, that our results may apply only to the conditions we have stated. A much slower rate, for instance, may reveal a different state of affairs so far as the value of the two methods is concerned. An investigation of this question is now in progress.

One group of subjects practiced each hand separately until six sequences could be played in succession in perfect time

and correct order. Then the two hands were practiced together until the same standard of perfection was attained. This standard of mastery was, indeed, arbitrarily chosen, but it did insure the formation of rather well established habits, both before simultaneous integration of them was attempted, as well as after. Simple as the task of mastering each hand separately seems, the habits were developed with no small amount of difficulty on the part of a large per cent. of the subjects. The range of time spent on each hand was large. Though a few subjects spent only two or three minutes, some worked considerably over an hour on each hand. The success of the subject, of course, was, in part, a function of the amount of previous experience he had had in the independent use of his fingers, keeping time, etc. Previous experience with the piano transferred well, as one might expect.

The other group of subjects attempted from the very first to manipulate both hands simultaneously and to keep time with the metronome, a very confusing task, even for those familiar with the technique of piano-playing.

The chart indicating the finger sequences to be used was supported on a frame and placed in front of the subject just back of the typewriters. Its position was constant, as was also that of the machines, which were placed far enough apart to prevent a collision of the moving carriages. The finger sequence to be used by the right hand was indicated by the top row of numbers on the chart; that by the left hand, by the bottom row of numbers.

All subjects practiced for periods of three minutes, separated from one another by rest periods two minutes in length. Time was recorded with a stop watch. No subject struggled for more than an hour at a time. If success was not achieved in an hour, and it usually was not, the subject returned on alternate days until the desired goal was attained. The motivation we attempted to sustain at as nearly a constant level as possible by frequent encouragement.

There were 45 subjects in each group and they were drawn from the undergraduate classes in education at the University of Texas. Since experience in the technique of piano-

playing is one factor determining the success of a subject, great care was taken that the average training in piano-technique of the two groups should be as nearly equal as possible. Each subject was questioned in regard to the length of time he had studied piano or had been playing with any regularity, and was placed in the groups accordingly. Such a standard of skill, of course, is crude, but it was the best one available. The experience factor we attempted to regulate further by rejecting all who had had any contact with typewriting.

As measures of learning we employed the time spent in actual work; the number of trials, *i.e.*, the total number of times the sequences were played by each hand; and, lastly, the number of wrong keys pressed. This method of recording as errors merely the failures to press the proper key and not the failures to keep time with the metronome makes the error score, taken singly, a very unrepresentative measure.

The data indicate that the part method tends to reduce considerably the total number of trials required for mastering the problem. Nine out of nine possible comparisons shown in Table I. reveal that the part-method groups have lower

TABLE I

AVERAGE TOTAL TIME, TRIALS AND ERRORS PER INDIVIDUAL FOR GROUPS LEARNING BY THE WHOLE OR PART METHODS WHO HAVE HAD VARIOUS AMOUNTS OF EXPERIENCE IN PIANO-PLAYING

No. of Years of Piano Study	No. of Cases		Av. Total Time per Individual (Minutes)		Av. Total Trials per Individual		Av. Total Errors per Individual	
	Whole	Part	Whole	Part	Whole	Part	Whole	Part
0 ...	6	4	150.65	96.77	1242.33	818.00	738.60	461.00
0– 11/12...	9	11	74.46	74.48	724.66	651.55	475.77	417.09
1-1 11/12...	3	5	76.38	40.55	755.33	361.00	357.67	347.40
2-2 11/12...	9	7	49.00	36.54	490.44	355.29	335.22	391.43
3-3 11/12...	3	6	42.66	36.16	437.44	345.33	343.66	348.00
4-4 11/12...	8	7	43.88	41.05	437.25	375.14	409.38	406.43
5-5 11/12...	2	0	40.63	....	465.00	.....	425.50	.....
6-6 11/12...	1	2	52.50	33.63	466.00	340.50	269.00	295.00
7-7 11/12...	2	1	33.45	17.29	335.00	185.00	335.50	151.00
8-8 11/12...	2	2	38.34	31.04	422.00	339.00	403.00	505.00

trial scores than the whole-method groups. In other words, the superiority of the part method, so far as the number of trials required in the learning is concerned, is not destroyed by varying the amount of piano study. While many of the averages in Table I. and also II., if considered singly, would

Table II

Average Time and Error per Individual per Trial of Groups Learning by the Whole or Part Methods who have had Various Amounts of Experience in Piano-Playing

No. of Years of Piano Study	No. of Cases		Av. Time per Individual per Trial (Minutes)		Av. Error per Individual per Trial	
	Whole	Part	Whole	Part	Whole	Part
0	6	4	.2422	.1970	.5594	.4840
0– 11/12.....	9	11	.2046	.1794	.7003	.7473
1–1 11/12.....	3	5	.2077	.1805	.4816	.9691
2–2 11/12.....	9	7	.2025	.1578	.6905	1.0910
3–3 11/12.....	3	6	.1944	.1760	.7437	.9096
4–4 11/12.....	8	7	.2044	.1778	.9543	1.1638
5–5 11/12.....	2	0	.1737	.....	.9437	.....
6–6 11/12.....	1	2	.2253	.1636	.5773	.8895
7–7 11/12.....	2	1	.2027	.1490	.9404	.8162
8–8 11/12.....	2	2	.1799	.1749	.9734	1.4852

have no significance because of the small number of cases upon which each is based, nevertheless, the uniformity of the results gives one faith in the validity of the tendency indicated. When, furthermore, the scores of all piano and non-piano groups are pooled and a classification made only on the basis of the method of learning, the superiority of the part-method group, provided one considers merely the total trials,

Table III

Average Time and Error per Individual per Trial of Groups Learning by the Whole or Part Method

The scores of the piano and non-piano groups have been pooled.

Measure	Whole	Part	Difference	P.E.$_{diff.}$
Av. time per individual per trial.....	.2066 ± .0027	.1754 ± .0080	.0312	.0084
Av. error per individual per trial.....	.7462 ± .0219	.9070 ± .0309	.1608	.0378

becomes more apparent. (See Table IV.) The difference between the averages of the two groups is 2.307 times the sigma of the difference. This means that the chances are 75 to 1 in favor of the part method as a trial-saving device.[1]

TABLE IV

AVERAGE TOTAL TIME, TRIALS AND ERRORS PER INDIVIDUAL FOR GROUPS LEARNING BY THE WHOLE OR PART METHOD

The scores of the piano and non-piano groups have been pooled.

Measure	Whole	Part	Difference	P.E.$_{diff.}$
Av. total time per individual....	66.66± 4.24	51.46± 3.86	15.26	5.73
Av. total errors per individual ..	442.37±28.92	390.96±22.64	51.41	36.73
Av. total trials per individual...	630.58±36.98	466.06±30.45	164.52	47.93

The time scores show tendencies not unlike the trial scores. In eight out of the nine possible comparisons indicated in Table I., the total time spent in mastering the task is less for the part-method groups than for the whole. According to the data in Table IV., the chances are 20 to 1 in favor of the part method as the more economical procedure, so far as time is concerned.

TABLE V

AVERAGE TOTAL TIME, TRIALS AND ERRORS PER INDIVIDUAL FOR GROUPS LEARNING BY THE WHOLE OR PART METHODS WHO HAVE NOT STUDIED PIANO[2]

Measure	Whole	Part	Difference
Av. total time per individual.....	150.65	96.77	53.88
Av. total errors per individual.....	738.60	461.00	277.60
Av. total trials per individual.....	1242.33	818.00	426.33

That this saving in total time is not merely a result of the saving in trials, but is also dependent upon an increase in speed, is revealed by Tables II. and III. and by the speed curves shown in Fig. 1. In nine out of nine possible comparisons shown in Table II., the average time per trial of the part-

[1] See W. A. McCall, 'How to Measure in Education,' New York: The Macmillan Co., 1922, p. 404 ff.

[2] The P.E.$_{diff.}$, is not given because the number of cases upon which the averages are based is so small the P.E. can have no significance.

method groups is less than that of the whole-method groups. In other words, their speed is greater. Since, according to Table III., the difference between the average time consumed per trial by the two groups is 2.474 times the sigma of the difference, the chances are 160 to 1 that the difference is a reliable one and in favor of the part method.[1]

TABLE VI

AVERAGE TOTAL TIME, TRIALS AND ERRORS PER INDIVIDUAL FOR GROUPS LEARNING BY THE WHOLE OR PART METHOD WHO HAVE STUDIED PIANO

Measure	Whole	Part	Difference	P.E.$_{diff.}$
Av. total time per individual....	53.74± 3.30	47.04± 3.78	6.70	5.01
Av. total errors per individual ..	391.66±24.35	384.12±21.04	7.45	32.18
Av. total trials per individual...	536.46±31.37	431.76±29.60	104.70	41.13

The speed curves (Fig. 1) reveal that the part-method group, when the simultaneous integration is attempted, has a speed considerably greater than that exhibited by the whole-method group in the initial stages of the learning, though far from the speed which each hand separately had previously attained. Not only does Curve *C* begin, then, at a higher level than Curve *D* but it rises more rapidly than does Curve *D* and reaches a higher level before it flattens to any extent. In other words, progress is made more rapidly to a higher speed level by the part-method group and the speed goal, furthermore, is sooner attained. While practice on parts,

[1] In computing the average time per trial, the number of trials, in the case of the part-method group, was derived in terms of the formula: (right-handed trials + left-handed trials + two-handed trials); in the case of the whole-method group, in terms of the number of two-handed trials. In computing, on the other hand, errors per trial, the number of trials, in the case of the part-method group, was derived in terms of the formula: (right-handed trials + left-handed trials + 2 × two-handed trials); in the case of the whole-method group, in terms of the formula: (2 × two-handed trials). Our justification for this procedure is as follows: The simultaneous playing of the sequences with two hands, other things being equal, should consume no more time than the performance of a single hand, since the metronome determines the rate at which the keys are struck whether with one or two hands. The situation is somewhat different for errors. Since twice as many sequences and, hence, twice as many keys are struck in a dual- as in a single-handed performance, there are twice as many opportunities for making errors. Consequently, each sequence should be taken into consideration.

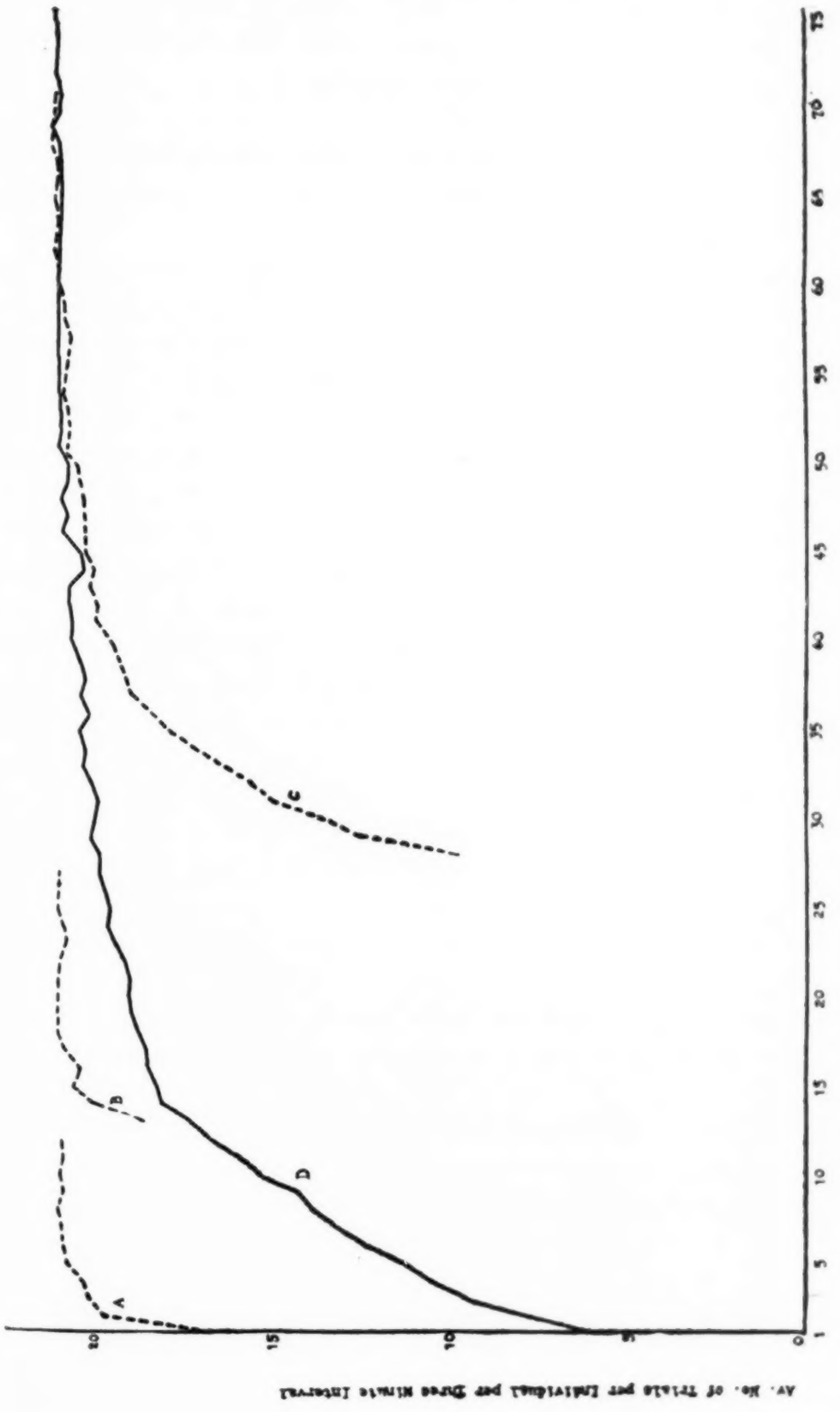


FIG. 1. Speed Curves. The unbroken line represents the acquisition of speed by the whole-method group; the broken lines, the speed of the part-method group. *A* is the learning curve for right hand alone before simultaneous integration is essayed. *B* is the curve for the left hand alone. *C* represents the progress in speed during the integration of the previously mastered parts into the more complex whole.

then, does not insure freedom from difficulty in the simultaneous performance of these parts, much that is beneficial does transfer and the learning is speeded.

It was noted that those individuals who trained first on the smaller complexes stressed speed when the simultaneous integration was undertaken, whereas those who dealt with the more complex situation from the start, seemed in the early stages of the learning to be not so vividly conscious of the speed phase of the problem. Consequently, a speed set far below that demanded was established before the subject was aware of it, and this set was broken down, as a rule, with more difficulty than most of the unfavorable position habits.

It is interesting that, though the part method accomplishes a saving in time, its effect upon errors is not so favorable. In Table IV. we see that the part-method group has a somewhat lower total error score than the whole-method group but the difference between the averages of the two groups is not a very reliable one. Furthermore, in only five out of nine possible comparisons based on Table V. do the part-method groups have the lower averages. Hence, we can feel no great confidence that a difference actually exists. Tables V. and I. do suggest, if an average based upon four or six cases can have even suggestive value, that perhaps a saving in total errors is more likely to occur in the non-piano than in the piano groups.

The scores for errors per trial indicate that the whole-method group is the more accurate of the two. In seven out of nine possible comparisons based on Table II., the whole-method groups have the better accuracy scores and the data in Table III. reveal the chances in favor of the greater accuracy of the whole-method group to be 6,700 to 1. This result we might expect, because the time in which the learning is accomplished is extended considerably beyond that of the part-method group and the error elimination process, hence, is not so condensed.

For information in regard to the question of whether this difference in the accuracy of the two groups exists at all stages of the learning we must turn to the curves of Fig. 2.

FIG. 2. Accuracy Curves. The unbroken line represents the elimination of errors in the whole-method group; the broken lines, the same, in the case of the part-method group. *A* is the curve for the right hand when working singly; *B* is the curve for the left hand. *C* represents error elimination during the integration of the previously mastered part reactions into the more complex whole.

The curves reveal that the accuracy which obtains in the case of the part-method group, when the simultaneous integration of the already perfected part reactions is attempted, is about the same as that attained by the whole-method group after three minutes of practice. The errors are, however, more rapidly eliminated by the part-method group than by the whole. It is interesting, furthermore, that only about nine minutes of practice by the whole-method group is necessary to bring the accuracy to the initial accuracy that obtains in a single-handed attack on the problem; but whatever errors are made in these single-handed attacks disappear very quickly.

Another phase of the data which casts some light upon the question of the relative value of the whole and part methods is that revealing the influence of training in piano-playing upon the success of the groups. Tables IV., V. and VI. indicate that experience with the piano not only reduces the total time, trial and error scores in both the whole- and part-method groups,[1] but it also reduces the difference, absolute and relative, between the two groups. This suggests that the advantage of the part method over the whole is greatest when the task is most unfamiliar and difficult.

[1] The correlations of total time, trial and error scores with the number of years of piano study, in the case of the part-method group, are $-.87$, $-.83$ and $-.19$, respectively; in the case of the whole-method group, $-.84$, $-.87$ and $-.66$, respectively. The formula used in computing the correlations is $p = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$.

TO WHAT EXTENT IS MEMORY MEASURED BY A SINGLE RECALL?

BY WARNER BROWN[1]

It is a matter of common observation that items such as names or dates which evade recall at one time can subsequently be recalled. Ballard[2] showed in a very extended series of experiments that the number of items given in a second recall is often greater than the number given just after learning, when there has been no learning in the interval. Huguenin[3] reports the same phenomenon. Myers[4] reports that intervals of one half-hour were not always better than longer intervals between observation and recall. Gates[5] states (p. 46) that some children gave more in recall after 3 or 4 hours than immediately after learning; but he discarded such data. Norsworthy[6] reports that many learners were able to give more vocabulary-words on an unexpected test after 30 days than at the close of the learning period. The list of similar references in the literature could easily be extended. The existence of the phenomenon is well attested.

The following experiments were devised to bring out this phenomenon and, if possible, to subject it to analysis. These experiments were planned upon two hypotheses. The first hypothesis is that a test of recall does not give an exhaustive measure of all the items remembered. Some items are remembered which are not at that particular moment recalled. Chance factors which vary from moment to moment affect the recall; and at any moment, due to adverse factors, the number of items recalled is inferior to the possible number which might be recalled (*i.e.*, the number retained). The

[1] From the Psychological Laboratory of the University of California.

[2] *Brit. J. of Psychol.*, 1913, Mon. Suppl., Vol. 1, No. 2.

[3] *Arch. de psychol.*, 1914, 14, 379–383.

[4] *Arch. of Psychol.*, 1913, No. 26.

[5] *Arch. of Psychol.*, 1917, No. 40.

[6] *J. of Educ. Psychol.*, 1912, 3, 216.

second hypothesis is that in any second recall the learner has the benefit of practice or recitation in the first recall. Ballard seems to have ignored this factor although it was well established by the work of Witasek,[1] Katzaroff [2] and others, and has since been confirmed by Gates [3] and by Kuehn.[4]

If the first hypothesis is valid it follows that a second recall would by mere chance give a list of items somewhat different from the first list. But according to the second hypothesis the items of the first list will be established by recitation. The second list will therefore be more extensive (under suitable conditions) than the first list because it contains those items which are fairly sure of recall in any list, plus a certain number of items subject to chance which failed to appear in the first but do by chance appear in the second, plus in addition items which by chance appeared in the first list and might not have occurred in the second, but which do occur in the second because of being reviewed or recited in the first recall.

In order to avoid rapid forgetting between the two recalls it seemed at first wise to take material for experiment which had not been learned recently. A list of the States of the Union seemed to meet this requirement. Students in an elementary course in psychology were asked to write in five minutes, the names of as many states as they could remember. A half-hour later they were asked, without warning, to repeat the performance. Lists were scored from 194 persons. The results, as tabulated below, show a net gain per list of more than 3 states and show that this gain when analyzed conforms to our two hypotheses. Over 13 per cent. of the items of the second list had not been given in the first, while only 5 per cent. of the first list was forgotten in the second recall.

Number of states remembered first test	36.31
Of these there appeared in the second test	34.37
And new items appeared in the second test	5.29
Making a total for the second test	39.66

[1] *Zsch. f. Psychol.*, 1907, 44, 161–185; 246–282.
[2] *Arch. de psychol.*, 1908, 7, 225–258.
[3] *Arch. of Psychol.*, 1917, No. 40.
[4] *Zsch. f. Psychol.*, 1914, 68, 396–481.

The partial success of this experiment led to a trial of 48 unconnected common words. This list partly obviates the criticism that students once started thinking about the list of states will continue with that task and so think up forgotten states during the interval preceding the second recall. It is possible, however, that the experience of the states experiment may have led some of the subjects to prepare for a second recall also in the case of the disconnected words. The second experiment took place two weeks after the first, with 193 subjects most of whom were the same as in the first experiment.

The words were taken from the key list of the Berrol memory system and were presented as follows: Each word was pronounced, then read in a short defining sentence, then pronounced again. Thus: "*Hat*, he had a *hat* on his head; *hat*." After the whole set had been presented thus, the list of 48 words was read through again. Five minutes were allowed for recall. The second recall was made after a half-hour of unrelated lecture.

Number of words recalled in first test	25.48
Of these there appeared in the second test	22.44
And new words appeared in the second test	4.33
Total words in the second test	26.77

The net gain of 1.29 word per person is again ascribable to variability in the lists and to the fact that recitation will tend to fix the chance items of the first list and carry them over to the second list.

This group of facts is readily understandable on the assumption that the list of items which can be recalled at any given time is determined in part by association cues or stimuli which only happen to be present at that time, but which are adequate to insure the recall of several weak items. At another time a somewhat different set of cues will be present and a somewhat different list of items will be recalled. But this is not generally observed because (*a*) the process of forgetting tends to lop off the weaker items of the list and prevent their chance occurrence after a lapse of time; and (*b*) the chance items of the first list tend to persist on account of their recitation.

Some insight is gained into this situation by considering the history of the weakest and the strongest of the associations in question. In the case of States, the 10 are selected for study which were most frequently recalled on the first test and these are compared with the 10 which were least frequently recalled.

Among the strong [1] associations only 2.6 per cent. of those once recalled are later lost. Of the weak ones 8.9 per cent. are so lost. But when we look at the cases of strong associations at first omitted by some subjects, we find that 78.1 per cent. of them are picked up on the second test, whereas for the weak associations only 35.1 per cent. of the first omissions are included in later lists. In other words, if subjects omit associations which we know to be strong, almost all of them develop these associations in the second list. But of the much larger number of subjects who omit what we know to be weak associations, fewer are able to develop these weak associations in their second lists.

TABLE I

STATES MOST LIKELY AND LEAST LIKELY TO BE RECALLED

	Strongest 10	Weakest 10
Recalled, first test, per person	9.25	5.52
Forgotten, first test, per person	.75	4.47
Recalled, then lost	.24	.49
Forgotten, then recalled	.58	1.57
Per cent. of first recall later lost	2.6	8.9
Per cent. of first forgotten later recalled	78.1	35.1

The second list is made up of (*a*) those items which belonged to the first list, minus some forgetting (which is mostly of weak items), and of (*b*) items actually retained which escaped the first recall (and these are mostly strong items which can survive the lapse of time).

A study of the 10 strongest and the 10 weakest of the 48 disconnected words shows the same situation in every respect.

[1] An association is called 'strong' if it appears frequently, *i.e.*, if it occurs to many persons including, necessarily, some of those who recall only a few items. *Cf.* Brown: 'Incidental Memory in a Group of Persons,' *Psychological Review*, 1915, 22, 81–85.

In the case of the States the associations are all old and all presumably of about the same age. But in the case of the words, the factors of primacy and recency offer convenient points of attack. The first 10 words in the list show some

Table II

Words Most Likely and Least Likely to be Recalled

	Strongest 10	Weakest 10
Recalled, first test, per person	7.72	3.25
Forgotten, first test, per person	2.28	6.75
Recalled, then lost	.48	.72
Forgotten, then recalled	.74	.88
Per cent. of first recall later lost	6.2	22.1
Per cent. of first forgotten later recalled	32.4	13.0

advantage over the last 10 from the very start. This advantage is increased in the second recall in which the first 10 words show a net gain of .071 per word per person, while the last 10 show a net loss of .031 per word per person.

Table III

Comparison of First 10 Words with Last 10 Words

	First 10	Last 10
Recalled, first test, per person	6.59	5.19
Forgotten, first test, per person	3.41	4.81
Recalled, then lost	.31	1.00
Forgotten, then recalled	1.01	.70
Per cent. of first recall later lost	4.7	19.3
Per cent. of first forgotten later recalled	29.6	14.5
Recalled, second test	7.30	4.89
Forgotten, second test	2.70	5.11

With respect to words once recalled but forgotten before the second test can be given, the first 10 show a marked immunity. The average person gives only .31 such words among the first 10, *i.e.*, only 4.7 per cent. of the first ten words are forgotten after they have once been recalled. But of the last 10 (recency) more than three times the absolute number, or 19.3 per cent., suffer this fate. On the other hand, if we consider the words at first omitted but later recalled we find that primacy shows a larger absolute number of cases than recency as well as a larger proportion of recoveries of this type.

Recency can be recognized as a factor responsible for the occurrence of words in the first list while in the second list recency has waned and words are given which although not found in the first list, have strength because of their position near the beginning of the list.

Conclusions

1. Two tests of memory do not agree because neither recall is a final measure. Each contains items by chance and each omits items by chance. In this case it can be seen that one circumstance (recency) is more influential at one time and another circumstance (primacy) at another time.

2. A second recall may contain more items than the first because repetition tends to fix the items of the first recall while chance tends to introduce new items into the second recall in addition to those retained from the first recall.

What has been said is not meant to prejudice the case against other explanations of the phenomenon which Ballard called "reminiscence." It is probable that perseveration and the factors which operate in the consolidation of dispersed learning and in the upbuilding of motor habits in the intervals between practices, as well as the unconscious supplementation of partially learned fragments, should all be considered together with the points urged in this paper.

A SIMPLE DEVICE FOR RECORDING INSPIRATION-EXPIRATION TIME

BY H. K. NIXON

Psychological Laboratory, Columbia University

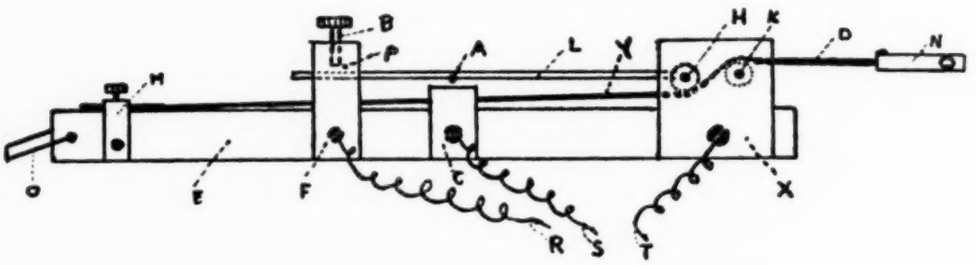
In the course of ordinary experimental work it is sometimes desirable to take a simple record of the number of breaths per unit of time or of the ratio of inspiration time to that of expiration. This can be obtained, of course, in the usual way with an ordinary pneumograph connected with a tambour which records its movements on a smoked drum. The reading of the record, however, involves considerable time and is hardly possible for experiments lasting for several hours. The simplest technique to date has probably been that of Burtt,[1] who devised a special attachment to be placed on the tambour to close an electric circuit during inspiration and open it during expiration, this change being recorded in any way desired. He found the apparatus satisfactory, although it would seem that the mechanism which he used on the tambour for making and breaking the circuit was rather delicate and likely to be in need of adjustment rather frequently.

The device herein discussed has the advantage of doing away with the pneumograph and tambour and the connecting rubber tube and substituting a simple mechanism to be fastened directly to the chest of the subject. There is no need for any adjustment and once in place it will function for hours, at the same time allowing the subject comparative freedom of movement about the room. It has been used with success to record the respiratory movements during sleep, and because of its small size affords a minimum of distraction to the subject. Since it is electrically operated it is possible to make the record either on smoked paper with an

[1] Burtt, H. E., 'A Pneumograph for Inspiration-Expiration Ratios,' *Psychol. Bull.*, 15, 1918, 325–328.

ordinary signal marker or with an ink pen on white paper or, if sufficient for the purpose, simply with counters which record the number of breathing changes.

The apparatus consists essentially of a mechanism for making and breaking an electric circuit at each change in the breathing cycle and some recording device such as those mentioned above to keep the record of such make and break. The first part of the outfit is shown in the accompanying diagram, which represents a side view of that part of the



apparatus which is fastened around the subject. The base, *E*, presses against the breast and is held in place both by a cord which passes from *O* around the body and is tied through *N*, and by a hook (not shown in the diagram) which is fastened to the base at *C* and which hooks into the clothing, thus preventing the apparatus from sliding downward on the chest. The base, *E*, is made of compressed fiber and is four inches long, half an inch wide and a quarter of an inch thick. Astride of it are fastened three brass mountings, *F*, *C* and *X*. The mounting *X* is pierced from side to side by two small shafts, *H* and *K*, upon which are freely turning brass spools which are housed in the mounting, *X*. The spool on *H* has soldered to it the thin strip of brass, *L*, which is cut very narrow, so as to be extremely light. It might be better to use aluminum here, as absence of extra weight is very desirable at this point, but in the apparatus as constructed in the Columbia Laboratory brass has been found satisfactory for all practical purposes. *L* extends from *H* in such a manner as to pass over the mounting *C* and under the mounting *F* and to be restricted in its movement up and down by the platinum tipped set-screw contact, *B*. At *A* is also soldered a bit of platinum to serve as a contact point.

Passing over the spool, *K*, and under the spool, *H*, is a length of rubber elastic cut to slightly less than half an inch width. This rubber is fastened to one end of the fiber base at *M* by a single clamp, and to its free end is fastened a brass eyelet, *N*. From *N*, as mentioned above, a cord passes around the subject's body and is tied to the other end of the fiber base by the swivel at *O*. Leading from *F*, *C* and *X* are the insulated wires, *R*, *S*, and T which run either to electric counters or to electrically motivated writing devices.

The operation of the device is very simple. The cord is placed around the subject's body and pulled taut until the rubber, *D*, is stretched to some extent. Then, with each inspiration or expiration, the rubber tends to adapt itself to the change in tension. This causes a portion of the rubber to be drawn over the spools *H* and *K*. By friction some of the motion of the rubber is imparted to the spools causing them to rotate; in the case of the spool *H* this results in the rise or fall of the lever arm, *L*, depending upon the direction of the breath. Thus in the case of inspiration the arm is pulled down and completes a circuit from *T*, through *H* and *A* and *C* and *S*. When the change to expiration takes place the direction of the movement of the rubber is reversed, thus turning *H* in the opposite direction and causing the arm, *L*, to flip up at once, breaking the first circuit and creating one through *T H P F R*. This movement of the lever arm takes place with a very small change in expansion or contraction of the chest and makes possible the recording of even short gasps and other slight changes in breathing. In practice it has been found that the sensitivity of this part of the apparatus can be increased by fastening a piece of linen tape to the portion of the rubber from *N* to the point *Y*. Tire cement can be used for this purpose. The effect of the linen backing is to confine the stretch of the rubber to that portion lying between *Y* and *M*.

In the Columbia Laboratory it has been found convenient to record inspiration-expiration ratios by this means over a period of five or six hours. In this case two Hollerith counters were used. One was put in circuit *T H L A C S* and in the

same circuit was a vibrating reed beating fifths of a second. Since this circuit was closed during inspiration, every five clicks of the counter meant one second of inspiration time. Knowing the total time, it was thus easy to calculate the time of expiration and the ratios. The second counter was placed in the circuit *T H P F R.* Since it was not in circuit with the reed it responded only once to each breathing cycle and thus served to show the total number of complete breaths per period. A graphic record was also possible at the same time by placing in either circuit an electrically motivated marker.